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350.

(Vol. XVI.—February, 1887.)

SPECIFICATIONS FOR THE STRENGTH OF IRON BRIDGES.*

By J. A. L. WADDELL, M. Am. Soc. C. E.

READ FEBRUARY 2D, 1887.

WITH DISCUSSION.

One of the most difficult and unsatisfactory tasks which come within the province of the civil engineer, is the preparation of specifications for bridges.

Bridge designing, when done scientifically, is an extremely complicated matter, and there are many circumstances connected therewith which are dependent upon experiment, experience, and even guess-work.

On this account there is, as might be expected, a great variety of opinions concerning many points among those engineers who have made a specialty of bridge-work. This fact is made evident by comparing the general specifications of several of the leading specialists. Not only do they differ essentially on many important matters, but some neglect entirely considerations which in others are considered essential. The difficulty under which all the writers have been laboring is that each has

* See paper on Specifications for Strength of Iron Bridges, by Joseph M. Wilson, M. Am. Soc. C. E., Transactions of the Society, Vol. XV, No. 335, page 389, June, 1886.

been working almost entirely alone, notwithstanding the fact that the finished specifications of the others are at his disposal.

Now, as any one change in a set of specifications will involve a number of others before everything can be brought into harmony, it is clear that, unless there be good opportunities for discussion by the various writers, a wide divergence must result. In submitting his specifications to the American Society of Civil Engineers for discussion, Mr. Wilson has done a good deed for the profession, and it is to be hoped that the Members of the Society will take advantage of the opportunity thus freely offered. If the subject of general specifications for bridges were thoroughly discussed in every particular by every Member of the Society who has made bridges his specialty, it would be possible to prepare a new set of specifications which would embody all the good points of all previous ones, and be as perfect as the present state of our knowledge will permit. If this desideratum were attained, the status of American bridge-building would be so improved as to stand out in even more vivid contrast than it does to-day with the crude and antiquated methods which are still employed by British engineers; and the result would be that for some time to come the majority of the most important structures required thoughout the world would be manufactured in the United States. Bridge-building in this country has become such an immense business that the home demand for structural iron-work is becoming less than the supply, so in order to keep their shops full, the American manufacturers will soon have to turn their attention to competition in foreign countries.

Australia, India, Japan, and even China, are all good fields for this enterprise, and it is not impracticable to compete in England.

The principal objection which can be raised against Mr. Wilson's specifications is their inconvenience, necessitating the use of a rather complicated formula, which involves the determination of extreme stresses in order to find the proper intensity of working stress. Would it not be much better to give, say by diagrams, the intensities of working stresses for each kind of bridge member for all practical cases?

"General specifications" are usually made to cover too extreme cases. Would it not be better to make them cover ordinary cases only, leaving extraordinary cases to be dealt with by special specifications?

For instance, cantilevers, very long spans and braced piers cannot be conveniently designed by specifications which were prepared especially

for ordinary spans. In the first two cases it may be advisable to use material of more than ordinarily high ultimate strength and elastic limit in order to reduce the dead-load; and in the last case the conditions affecting the design are or should be very different from those affecting that of the spans. Simplicity, rather than an elaborate system built up on deeply scientific methods, should be one's object in preparing specifications. As an example let us take Mr. Wilson's specifications for beam-hangers, viz.: "Floor-beam hangers must have an additional section of twenty-five per cent. above that given by the before mentioned limiting stresses," which limiting stresses are determined by Launhardt's complicated formula. Would it not be much simpler and more satisfactory to say that "floor-beam hangers shall be proportioned for a working stress of ——— tons per square inch," covering thereby not only the effect of extremes of stress, but also that of impact?

As another example, let us take Mr. Wilson's method of proportioning top flanges of girders. It reads as follows: "In all cases for compressed flanges of beams or girders (subject to transverse stress), the permissible working stress in such flanges shall be computed by Rankine's formula:

$$c = \frac{a}{1 + \frac{l^2}{5\,000\,w^2}}$$

Where a = permissible stress previously found.

c = allowable working stress per square inch.

l = unsupported length in inches.

w = width in inches.

In no case shall a stress greater than that for a length equal to twelve times the width be used."

Let us see what are the various steps to be taken in determining this value of c, which most engineers assume to be constant and equal to four tons. First we must find the ratio of dead and total loads for the girder, and determine a by Launhardt's formula, then assuming w, substitute in the equation, and find the value of c. Of course the amount of work involved may be reduced greatly by the use of two diagrams. But is all this refinement really necessary? When we consider that the percentage allowed for impact is the result of mere guesswork; that the web is assumed as not helping to resist bending when it really does so assist; that the effective depth of the beam is inexact; that the rivet-

holes may or may not be counted in the effective sectional area according to the opinion of the designer; and that the "secondary strains," as pointed out by Bender in his "Principles of Economy in the Design of Metallic Structures," are decidedly great, we may conclude that the last question may be answered in the negative. There is hardly a single particular in plate-girder designing where theory will apply—the web thickness cannot be analytically determined; the proper spacing of stiffening angles is a mere matter of opinion; the rivet-spacing in the flanges, when the effect of concentrated loads is considered, is best arranged by making it uniform from end to end of span, and the effect of the web in stiffening the upper flange is an unknown quantity.

Is it not much better, then, to proportion beams and girders by a few empirical rules, which are the result of experience and good judgment, rather than by a deeply scientific method which is based upon assumptions that are known to be, if not absolutely incorrect, at least very loose approximations?

Again, the formulas of Launhardt and Weyrauch were established for tension only. Is it then advisable to adopt their complicated method of determining the intensities of working compressive stresses until it be proven that their deductions apply to compression as well as tension? Mr. Benjamin Baker appears to think that they do not so apply.

Again, in pin-proportioning, can any designer spare the time to find the value of a, multiply it by 1.5, and find the resisting bending movement of one or more pins for the intensity thus determined? The amount of material saved by using this instead of the ordinary method of employing tables calculated for constant intensities of working bending stresses, would not be worth as much as the time that the computer would lose in making such elaborate calculations.

When specifying three classes of engine loading for the same bridge, it would be well to state for each kind of member the cases in which each loading will produce the maximum effect—this would effect a great saving of time for computers.

In calculating the stresses in trusses, if one employ the latest and best method, as given in the third edition of Burr's "Stresses in Bridge and Roof Trusses," it would be incorrect to neglect the weight on the first pair of wheels.

Mr. Wilson, in proportioning top chords, evidently figures each panel length as if hinged at the ends. Although this is not in con-

formity with general practice, it appears to be correct. If one figures them as hinged, it will be well to make them pin-connected, and avoid entirely field-riveting for top chords and batter braces.

The allowance of five tons for initial tension on each adjustable rod does not appear to be either scientific or practically correct. Surely it takes less initial tension to properly tighten a one-inch than it does to properly tighten a two-inch rod. The allowance which I have given several times in books and papers, is contained in the folowing table:

Diameter of Rod.	In Tension.	Diameter of Rod.	In Tension.
Inches.	Tons.	Inches.	Tons.
1	1.00	1¾	2.50
1⅛	1.25	1⅞	2.75
1¼	1.50	2	3.00
1⅜	1.75	2⅛	3.25
1½	2.00	2¼	3.50
1⅝	2.25	2⅜	3.75

These amounts are not certified to as correct, being the result of mere guesswork; but it is submitted that the method is more rational than that which allows five tons for any rod, irrespective of its diameter. Perhaps the allowance should be a certain amount per square inch, in which case the quantities in my table do not increase quite fast enough. A series of experiments by several experienced bridge erectors upon this matter would be very useful. The tensions measured by a dynamometer might be recorded by an assistant, without communicating their amounts to the men who do the adjusting.

It would appear that Mr. Wilson condemns the use of iron track stringers having no plate on the upper flange. Perhaps this is because he places the stringers more than five feet apart, center to center. If the stringers be placed directly under the rails, I see no reason for insisting on the use of a top plate. Concerning the proper position for stringers, I will have more to say in a subsequent communication to the Society.

I should like to ask Mr. Wilson why he allows the use of continuous spans in deck and not in through bridges. The objections to this arrangement which hold in one case, hold equally well in the other.

Is not a space of ten inches between track ties altogether too great to permit the passage of a derailed car?

In my opinion wooden outer-guard rails are not nearly so efficient as inner-guard rails of angle iron.

Concerning the best arrangement of ties and guard rails, I would like to call the attention of the Members of the Society to the floor system proposed in my "System of Iron Railroad Bridges for Japan," and invite their criticism thereon.

Mr. Wilson, in common with many late writers of bridge specifications, does not provide for a higher intensity of wind pressure upon empty than upon loaded bridges. It is to be noticed, though, that he specifies, in proportioning iron piers, a pressure of fifty pounds per square foot on the unloaded structure. If fifty pounds per square foot will buckle the windward bottom chord (as it will in most of the existing single-track bridges) it is useless to proportion the piers for this pressure.

In conclusion, I wish to observe that, in thus criticising Mr. Wilson's specifications, I do not intend to depreciate their value. They are, perhaps, as good as any others that have as yet been writen. The fact is that all existing specifications for bridges are far from perfect in many respects; and, as before stated, the only way to make them approach perfection is to submit them to thorough detailed discussion.

DISCUSSION.

JOSEPH M. WILSON, M. Am. Soc. C. E.—I think that the objection which Mr. Waddell raises as to the inconvenience of the application of my specifications is more apparent than real. The work is very much simplified by the use of tables, which we employ to a large extent.

One point in reference to the specification is that it is adapted to cover all general cases, cantilevers, long spans and braced piers as well as short spans; also work on buildings where the variations in live and dead loads are usually much greater than in bridges, and if a higher grade of material is desired, the necessary changes in the constants can readily be made. It is always feasible to make modifications to suit special requirements, and it seems to me that this is the only proper way to treat the subject. One should work from the general case to the special, not from special to the general. It is true that some details, such as floor-beam hangers, might be more simply treated as suggested

by Mr. Waddell; but after all, when the stress is obtained by formula for a certain case, the results can easily be used thereafter as a constant quantity.

Concerning the question of the labor of proportioning the top flanges of girders, the value of a is easily obtained from tables by merely dividing the minimum by the maximum, and this allows of any relative variations in live and dead loads from that of a girder having all dead load, as in certain cases in buildings, to that of nearly all live load.

For most cases of plate-girder work, the lateral bracing being placed at such distances apart as not to be more than twelve times the width of a plate, c is obtained by taking a fixed percentage of a for that value of $\frac{l}{W}$, as shown by tables. The formula is more particularly intended to apply for cases in which the value of $\frac{l}{W}$ much exceeds twelve, and where the ordinary column formula will not apply on account of the flange obtaining assistance from the web.

If Mr. Waddell's method of reasoning concerning the proportioning of beams and girders were carried out, it would tend very much toward reducing calculations to a "rule of thumb." The question is: Is it better to work by the nearest approximation to correct rules that one can obtain, or to work to no rules at all? Because these principles may not be absolutely exact, it does not follow that they are not more correct than mere guesswork.

That they are particularly intricate of application I cannot admit. Things which appear intricate by observation, very often are found in practice to be very simple, especially with the use of a few general tables. I argue that it is better to work to a system throughout, and if it is found inaccurate to modify it, rather than throw all system away. No one is more willing to modify than I am, when satisfied that there is something better.

Concerning the proportioning of pins, I have tables that give the bending moments for all proportions of pins, and all that it is necessary to do is to select the proper figures and multiply by $1\frac{1}{2}\,a$. This surely does not require a very large amount of time.

As to the various classes of engines, these specifications were framed for a particular road with certain kinds of engines, being originally made for only two classes, until the "M" engine came in as a later type, being found very heavy for cross-girders and similar parts. We

do not advocate the use of these engines for all roads, and in fact modifications would be advantageous for this road now; but they were adopted some years ago under the supposition that they were a sufficient advance over the actual service to cover some years of improvement. Our labors with them have been very much simplified by having their results tabulated. I am decidedly in favor of generalizing by the use of an assumed type of engine, or of loading, that will cover all cases in practice, even at the risk of increasing somewhat the weight and cost of the bridges. My fault in preparing this paper perhaps was that I did not start out to present a new specification of exactly what would be best in every respect for general use to-day, but I gave truthfully and exactly what was at that time standard for a particular railroad. It is difficult sometimes to change a standard at short notice, and what I wanted to do was to show the practice of several years, the results of which are visible in bridges which now exist. Were I to rewrite these specifications, I might improve them in a few particulars, not only from the consideration of just criticisms on my paper, but from later experience of my own, although I am satisfied with their main features thus far, and would not make any material changes.

The method of calculation adopted is by the use of panel loads, and the omission of the front wheels applies entirely to that. It is not in any way essential to the specification, but if the panel-load system is used, it is on the side of safety and simplifies the calculations.

The question of hinged ends in top chords has in the specification a saving clause, to which I would direct Mr. Waddell's attention. Cases occur when from the weight of the chord itself in adjacent panels, or something similar, the chord is incapable of bending in opposite directions on opposite sides of the point of support. In such cases I would not consider it as hinged.

As to initial tension, I would like to ask Mr. Waddell whether he proposes to increase the number of men on the lever for screwing up when he increases the size of the rod. The matter is more a function of the man than of the size of rod. The desire is not to make the small rods too small for practical use, and the rule gives an allowance for screwing up on the small rods for safety. It is more needed on the small rods than on the large ones. We know that all rods are screwed up beyond simple tightness, and that if they be small they are more easily over-strained than if large. We have no idea of putting on an

actual initial strain per square inch over all rods in proportion to their size.

Concerning the question of girders with no upper flange plate, I would refer to my previous reply (see Transactions, page 487, Vol. XV).

Mr. Waddell misunderstands my limitation in reference to continuous girders. It is not a question of deck and through bridges. Continuous girders are allowed in drawbridges and also in the upper chords of deck bridges as a girder carrying a floor between panel points. The permission does not refer to the whole truss as a truss.

Ten inches between track-ties is not too great to permit the passage of a derailed car, as I can testify by numerous instances; in fact I have seen cases where a train has crossed over the whole length of a bridge on the old-fashioned 8 by 14-inch white pine floor beams laid 2½ feet apart, center to center, and having longitudinal stringers under the rails, notched on. Very decided marks were left, it is true, by the wheels, but the cars got across all the same. A great point is to hold the floor beams or ties in place and prevent them from piling up, and the exterior wooden notched guard-rails do this, as well as acting to keep the train on the bridge. Inside iron guards can be used in special cases, such as on elevated roads, for exclusively passenger traffic; but if a brake block should fall between such a guard and the rail, a chance not by any means rare, particularly where freight trains are run, it may produce very serious results. An inner iron guard rail as ordinarily arranged will not prevent the ties from piling up, thus weakening the floor system and perhaps letting the train through.

The question of stability of iron piers under wind is dependent a great deal on the load on the bridge. When the bridge is unloaded the tendency to overturn is greater and the rule of the specification is intended to give greater stability to a lightly loaded pier.

I am glad to see discussions on the paper, and desire to profit by any experience that can be brought to bear on the subject, such experience, however, being always open to criticism as well as the original question.

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(Vol. XVI.—February, 1887.)

VIBRATION OF BRIDGES.

By S. W. ROBINSON, M. Am. Soc. C. E.

PRESENTED JUNE 26TH, 1885.

The object of this communication is to present to the Members of this Society a mass of facts and figures obtained in connection with the application of a so-called bridge indicator to railway bridges, and to mention some of the conclusions toward which those figures point.

EXTENT AND RESULTS OF THIS INDICATOR WORK.

The indicator was applied to thirteen different bridges of four different railways, resulting in one hundred and ninety-three indicator diagrams. Most of the diagrams were obtained from bridges of the New York, Pennsylvania and Ohio Railroad, though the first were obtained from a bridge on the Pan Handle Railway, and others later from several branches of the Pennsylvania system and the Baltimore and Ohio. The

NOTE.—This paper is prepared from advance matter from the report of 1884 by the author to the Commissioner of Railroads of Ohio on an investigation of the cumulative vibration of bridges made under sanction of Hon. H. Sabine, Commissioner.

aim in procuring an abundance of figures on the vibratory movements of bridges, was, if possible, to detect any extraordinary movements which might occur but rarely in the lifetime of a bridge, particularly cumulative vibration. In this the indicator itself surprised us with a discovery of vibratory movements due to a cause as startling as it was unanticipated, the cause being a combination of circumstances, including speed of train, car length, panel length, time of vibration of loaded bridge, rigidity of bridge flooring, etc. Besides this cause of cumulative vibration, the anticipated one due to unbalanced locomotive drivers found confirmation.

In this work no claim is made of discovery of new laws; on the other hand everything, as far as yet observed and studied, is traceable to previously well known and comparatively simple laws.

Need of an Indicator.

That a bridge is agitated as a train passes over it at speed, no one questions; but the precise character of the bridge movements during such agitation is a matter which cannot be determined by mere casual observation, because too rapid and complex. Some instrument which shall analyze the movements, separating them into horizontal and vertical components, and record them for subsequent examination, must be conceded to be one good means for determining those movements. This the bridge indicator above mentioned has done, copies of some diagrams from which are given in Plates V, VI, VII.

Previous Instruments and Use.

Movements of bridges under moving load have previously been recorded. The earliest instance that has come to my knowledge was that of J. T. Fanning, M. Am. Soc. C. E., who, in 1875, obtained diagrams by aid of a station near midspan, to which, and to the bridge, a pencil and a card were so attached that the motions of the bridge were marked down on the card, giving a diagram of the actual motions of the bridge. I understand that the taking of such diagrams has been the frequent practice of this engineer. In my tours of inspection of Ohio railways, I have found several instances of the determination of the deflection of bridges by means of a pencil attached to the bridge and a rod set on the ground or bed below, upon which the pencil could mark the deflection. In September, 1881, I procured diagrams similarly, as previously done by Mr. Fanning, but without knowledge of his experiments, copies of

which diagrams were published in the Ohio Railway Report for 1881 by H. Sabine, Commissioner of Railways for Ohio, being the first published diagrams of bridge motion that I am aware of. In these instances the card was attached to the station erected at midspan, and the pencil to the bridge, or *vice versa*, it being immaterial which. The diagram resulting from this device is a confused and knotted mass of lines, furnishing comparatively little information.

In the same railway report for 1881, the general character of a more complete and perfect bridge indicator was fully set forth, the same contemplating clock-work for uniformly moving a strip of paper before the two pencils, one of which marks the horizontal and the other the vertical movements of the bridge as the paper moves along. Some two years subsequent to this outlining of the complete instrument, a similar one was used by a Mr. Biadego on a three-span continuous girder bridge, notice of which was given in a foreign paper.

The numerous results found in tables given in this paper were obtained from diagrams taken between August 1st, 1884, and the end of that year.

Present Indicator.

The indicator used in these experiments might be briefly described as consisting of a heavy eight-day brass clock-movement, from which the escapement was removed, and in its place was put a small centrifugal governor, with a spring to counteract centrifugal force, and arranged so that, for a given position of the governor weights, pads were pressed against a disk, causing friction to absorb excess of driving power, and upon a shaft of which clock-work was attached a drum for moving the paper strip with two pencils, so arranged as to move lengthwise the drum, all being mounted on a base board and portable. In use the whole is clamped in position by a bolt passing through the base. The paper is first wound upon a separate drum held by slight friction on a pin. From this it is unwound as it is wound up again on the driving drum attached to the clock-work under the pencils. A fine slit along the length of the drums served well to secure the end of the paper as the latter was wound up; no great length was allowed to accumulate to enlarge the drum. The paper was speeded at about fifteen inches per minute. One pencil recorded the vertical movements and the other the horizontal. To secure the greatest freedom of pencil movement, the latter were secured on the ends of light bars about a foot long, the opposite ends of which were pivoted. The pencils thus moved in circle arcs, though for the small movement as compared with the radius, the lines of pencil movement were nearly straight and crosswise to the strips of paper.

The pencils were moved by cords, one going direct and the other going over a pulley to change its direction to a right angle, in order to bring

the pencil movements upon the same strip of paper. The cords were attached to the bars at 0.64 of their length from the pivot, so that the diagrams as taken were correspondingly widened; hence any ordinate or amplitude on a diagram must be multiplied by 0.64 to obtain the correct measure of bridge movement in inches.

A third and stationary pencil held by a spring was used to mark a reference line upon the same strip of paper.

Application of Indicator.

Several ways of using the instrument were tried, but reliable diagrams could only be obtained by placing it on the staging or support brought up from the bed below, so that it could be quiet, while the cords moving the pencils were tied to the bridge. When the indicator was attached to the bridge the jarring of the bridge was found to jostle the pencils too much, breaking the lines into dots, and the particular governor in the instrument used was sensitive to agitation. But there is evidently no reason why the instrument may not be attached to the support, and the cords to the bridge, instead of the reverse.

In one case the instrument was placed on the ground under the bridge, water not being under that span. But the best plan consisted of setting up a tripod, resting on the bed below, and reaching up through the bridge flooring without touching it, and extending to a convenient height and position for receiving the indicator. In one instance, on the double track Pennsylvania Railroad, there were two bridges, one for each track. Here the instrument was placed on one bridge, and the cords attached to the other, but the diagrams taken for trains moving over the bridge to which the indicator was attached were imperfect. Experience with this indicator, as mounted upon the bridge itself, would indicate quite conclusively that the harsh and fine cut tremor of the parts of a bridge during the passage of a train, is too severe for the good of any instrument that might be brought in contact with them.

Greater refinements of registry might be attempted than were carried out in these experiments. Pencils were used to do the marking, but they are faint. Diagrams made with an inking point would be much preferable for reproduction in printing. Also no special devices were employed for recording speed of train, as might have been done by electricity. It is to be regretted that the passage of every wheel of the train was not electrically recorded to aid in determining the relation of car lengths and time of vibration of bridge; also the down position of crank pin relative to panel.

The reference line pencil was used to record by hand and eye the revolution of drivers, and the time for a train to move from one signal to another at the ends of a given measured base; but automatic registry is much to be preferred, as far as can be, so as to leave the operator

free for making any notes he may desire about the train, such as number of engine, kind of cars and location, flat wheels, etc. Present experience shows that the indicator should be made to do all the recording possible. If another campaign were to be undertaken by myself, greater demands would be made of the indicator for registry with more or less of electrical attachments.

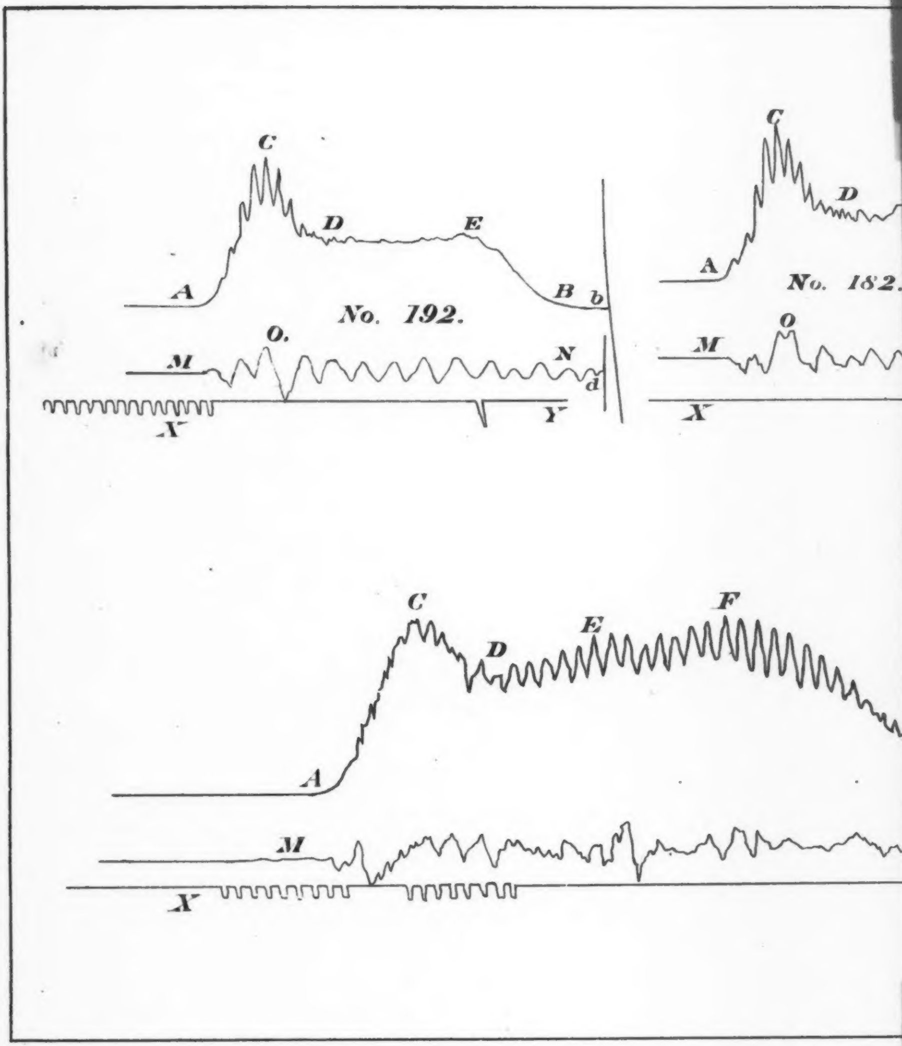
The Diagrams.

Accurate copies on Plates V, VI and VII give a good idea of the diagrams taken in this indicator work. Of the three lines traced as above mentioned, the upper one, $A\ B$, is that showing the vertical movements, and $M\ N$ the lateral movements, of the bridge during the passage of the train, the lower line, $X\ Y$, being the line of reference.

For all the diagrams the indicator was placed at the panel poi nearest mid-span, the cords connecting with pencils being always attached to parts of bridge near this point, and always to one truss only. In this way $A\ B$ is the record for the vertical motion, and $M\ N$ for the lateral, of the panel point of the one truss mentioned. Now, when a train approaches the bridge the indicator is started, making the straight lines at the left of A, M and X. But when the train strikes the bridge the bridge is disturbed, and the pencils respond accordingly, continuing so to do as long as any disturbance lasts, and that cannot be less than the time the train is on the bridge.

When the engine and train move upon the bridge, the latter is deflected on account of the load, and hence the pencil recording vertical movements responds to this deflection as well as to vibration, causing the very strong rise of the whole line $A\ C\ D \ldots . B$ above a straight line from A to B, a deflection of the bridge being here noted as a rise in the pencil. A smooth line drawn through the middles of the sinuosities of $A\ C\ D \ldots . B$ shows by the height of any point above a straight line, A to B, the statical deflection of the bridge at the corresponding time, and as the paper moves uniformly from A, where the train first touches the bridge, to B, where it leaves it, the deflection corresponding to any part of the train can be accurately located. While a smooth line through the sinuosities of $A\ C\ D \ldots . B$ answers to statical deflection, the sinuosities themselves answer to vibratory disturbances, the same appearing more or less regular, according to approach of bridge toward the condition of actual vibration. The above remarks relative to the record of the vertical movements of the bridge apply equally to $M\ N$ for the lateral movements, except that here we do not look for statical deflection unless the bridge has a curved track, or that wind is blowing while the train is passing.

The track was straight in case of all the diagrams shown on the plates.



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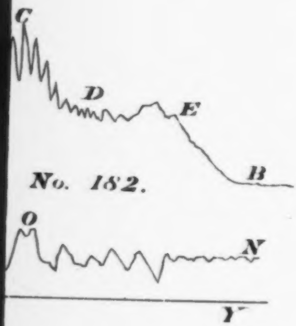

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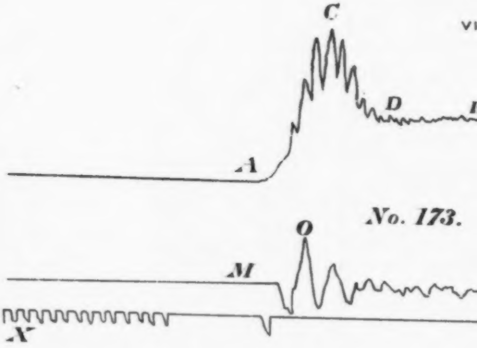

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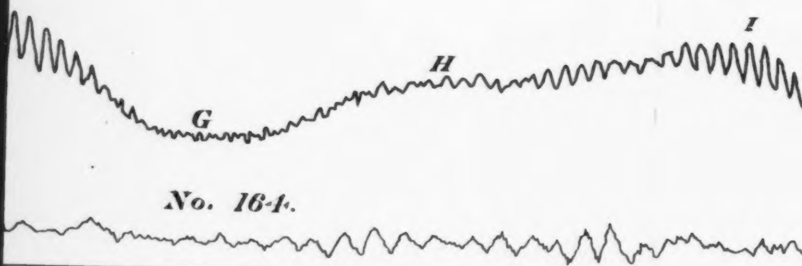

No. 164.

PLATE V
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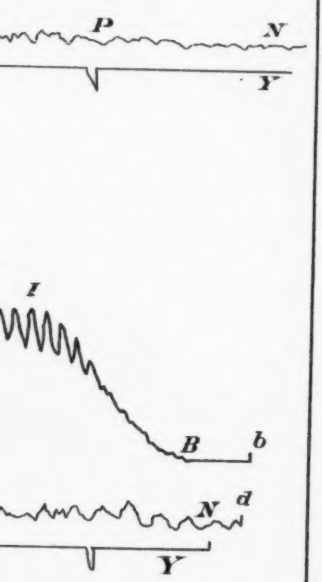

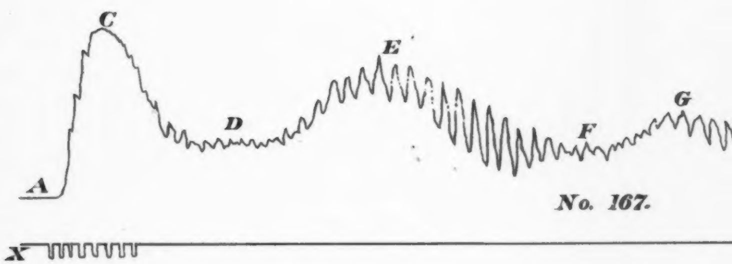

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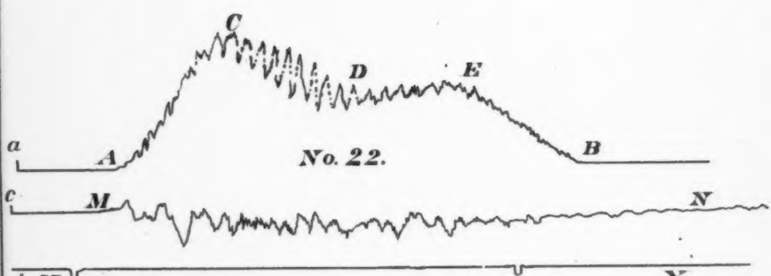

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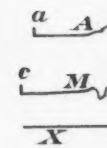
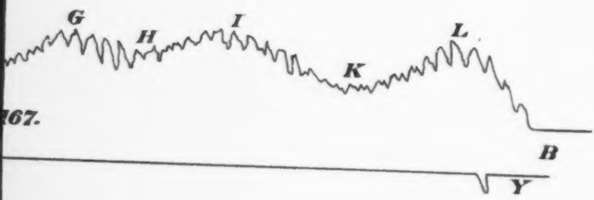
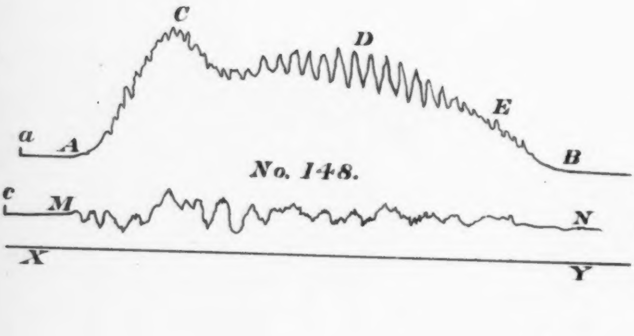


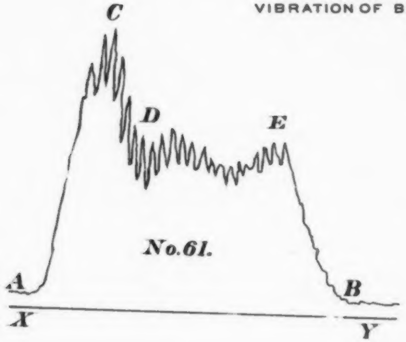


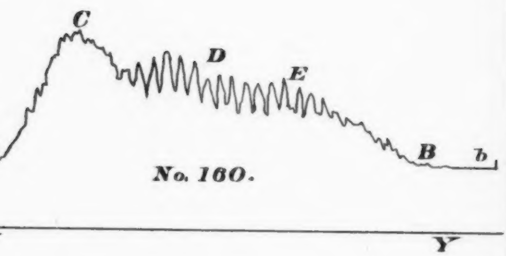
167.



No. 148.

PLATE VII
TRANS. AM. SOC. CIV. ENG'RS.
VOL. XVI Nº 351.
ROBINSON ON
VIBRATION OF BRIDGES.





To the left of A the lines are always smooth; but beyond B there may be sinuosities due to residual vibration of bridge. These are less apparent in $A\ B$ than in $M\ N$, though sufficiently numerous in each to give the time of vibration, lateral and vertical, of the unloaded bridge.

The diagrams 173, 182 and 192 differ very much from the others in appearance, owing to the fact that they are for passenger trains while the others are for freight. In the former the sinuosities at C are wide in amplitude, while the part $D\ E$ is comparatively smooth. On the other hand the freight diagrams are comparatively smooth at C and at places wide in amplitude of sinuosity beyond D.

The diagrams selected for the plates are among the more interesting ones, although others present like characteristics, as can be seen by consulting Table No. 2.

The speed of the paper when the diagrams were taken was from 15 to 15.5 inches per minute, and carefully noted at each setting of instrument; consequently the length of time any train occupied in going over a bridge can be measured from the diagrams. The length of 137 from A to B is 6.5 inches, so that the train was agitating the bridge about twenty-five seconds. The lateral movements of the pencils for A, B and M, N is in excess of the actual movements of the bridge by the ratio of 1 to 0.64. The points a and c give simultaneous positions of the two indicating pencils, and likewise for the points b and d.

The Tables.

Table No. 1 is a table of bridges, 1 to 13, noted in the order in which the indicator was applied to them. No results of indicator work are noted in this table, it being intended as an embodiment, in condensed form, of the leading particulars of the bridges.

In the column of stringers the term "I beam" means a solid rolled I beam; while "I section" means a beam built of a plate and four angle bars, and perhaps a cover plate, riveted together.

Table No. 2 is intended to include all results of value obtained from the diagrams, such as statical deflection ordinates; amplitude of vibration, vertical and lateral; number of vibrations per inch of diagram; inches of length of diagram for each train; number of revolutions of drivers per inch of diagram; and remarks.

The number of cars in column 3 was obtained from railway officials as far as possible, and checked by count taken at time of indicating bridges, though in some cases the number is stated simply from count. The number in column 4 is made out by aid of columns 12 and 15 for the cases of cumulative vibrations, on the supposition that the half-car lengths coincide with the time of vibration of bridge for such vibration. An agreement is corroborative of the fact of cumulative vibration for freight trains where

$$\text{Number of cars} = \frac{\text{number of vibrations per inch}}{2} \times \text{number of inches per train diminished by inches for engine.}$$

For instance, in diagram 61, averaging the vibrations 13.5 and 17 gives 15.25, and the length is 1.4 inches. These in the above give

$$\text{Number of cars} = 10.6 - 1.7 = 8.9 \text{ cars;}$$

a figure which agrees with the count within less than one car. In this a car is reckoned at 30 feet and an engine at 50 feet, making the engine about 1.7 cars.

The data in column 5 are made out from field notes and statements of railroad officials. Column 6 is made out entirely from information furnished by the railroads.

The speeds in column 7 were determined in three ways. See footnote to table.

Quantities in columns 8, 9, 10, 12, 15, 17, 18, and part of 16, were taken directly from the diagrams by measurement and count, a portion of 16 being found from columns 3, 5, 6, 15, and car length.

The percentages in column 11 were obtained by dividing half of the amplitude of vibration by the statical deflection of column 9, the latter being the height from the line *A B* to the mid-height of the sinuosities of the upper part of the diagram.

Column 13 was calculated by the aid of a theoretical formula published in the Ohio Railroad Report for 1881, with constants determined for each bridge.

Column 14 is calculated from twice the number of whole car lengths as divided by inches of diagram per train.

Columns 19 and 20 contain remarks as noted from the appearance of the diagram. Any remark applies to the line or bracket at which it is found. To determine whether the remark "cumulative" applies to the engine or to the following part of train, reference may be had to the eighth column, where any remark applying to the engine will be found at abscissas less than about 0.6 inch.

Table No. 3 exhibits, collectively, the results from those diagrams which record the more marked cumulative effects as due to the engine itself, and mainly due to unbalanced drivers.

In this table, columns numbered below 20 are to the same purpose as those of like number in Table No. 2. Column 22 is added to show what influence the floor beam distribution may have upon the vibration.

Table No. 4 is similar to Table No. 3, except it is intended for the train itself instead of the engine. A column for floor beams might have been added, but it would be so nearly identical with 14 that the latter may practically be taken for it, except in case of Bridge 6. See Table No. 1, where the floor beams are two feet apart or less.

Table No. 5 is intended to present points of interest as determined

by indicator respecting the statical deflection of bridges as due to a locomotive and first portion of following train; as due to uniform and ordinary train load; and also the time of vibration of bridges, vertical and lateral, loaded and unloaded.

Bridge Vibration and Oscillation.

I have applied the term vibration to the movements of bridges now being considered, instead of the term oscillation, for the reason that these movements exist, as to and fro motion, by reason of the elastic forces of the bridge itself. The movements of a long suspension bridge from the action of wind and gravity might be termed oscillation, because due to the action of forces existing outside of the structure. Vibration is usually much more rapid than oscillation. A glance at the diagrams is sufficient to show that many instances of real vibration have been recorded. The duration of one movement is too short for oscillation. These remarks, however, apply with full force only to such bridges as have all parts bound together so as to form a unit for elastic reaction, as in Pratt, Howe, Post and similar trusses. In a Bollman truss, for instance, where each floor beam is supported nearly independently of the others, and by tie-rods of different lengths and inclination, we find conditions very unfavorable for vibration. This view is supported by the few indicator records obtained from a Bollman bridge. This fact may serve to offset some of the unpopular features of Bollman bridges. But in the Pratt truss, to which the experiments were mostly confined, the upper and lower chords are joined firmly by bars into a whole, so that if one point is depressed by weight, the whole truss is sprung into an elastic curve. Such a structure may therefore be regarded as an elastic body, and susceptible of elastic vibration.

Vibration of Elastic Bodies.

Vibrations may be set up in an elastic body, 1st, by delivering upon it a severe single blow; 2d, by suddenly releasing it from a straining force; or 3d, by the successive application of a series of comparatively very small impulses, provided the impulses are applied at proper intervals of time. The most favorable condition is realized when the time intervals between successive impulses are equal one to another, and also equal to the periods of simple vibration, the impulses being applied to and fro with the movement of mass. A less favorable condition exists when the impulses are half as frequent as above, but applied all in the same direction. Other, though less favorable, conditions exist; but the present investigation has not shown that they take effect in bridges. Vibration from the third cause, slight at first, but eventually reaching a high intensity, may properly be called cumulative vibration.

A common hand-saw may serve to illustrate. Fasten the tip end in a clamp, or into a kerf sawed in a post, so that the blade will stand out horizontally and flatwise, with the handle at the free end. If now the handle be struck, or if it be pulled down some distance and released, the handle will move up and down repeatedly, illustrating the first and second cause of vibration.

To illustrate the third cause, or to obtain a cumulative effect, place the saw at rest, tie a thread to a tack and suspend it about eight inches from the hand. Now, dropping the tack upon the saw near the handle, a slight deflection may be observed due to the weight of the tack; as soon as the deflection takes place, lift the tack from the saw. When the saw handle has made its return and is ready to descend again, drop the tack on it once more; this adds a new impulse, and the vibratory movement will be greater than before. By continuing this the saw handle will soon gain an intensity of vibration that would hardly be expected by one who never tried it. But a more rapid increase of amplitude will follow the application of impulses in both directions, up when the saw goes up as well as down when the saw goes down, thus realizing the most favorable condition mentioned.

This illustration of cumulative vibration shows that the applied impulses must succeed each other at intervals which are in keeping with the vibration period, or that they must be "in time." To harmonize observed facts in bridge vibration with this principle, the time of vibration of the bodies concerned should be calculated, as of the bridge itself, the car on its springs, the engine on its springs, etc.

Calculation of Bridge and Car Vibration.

For a car, let the weight of the body and load be W, including all carried on the main car-springs. Let P be a force that will compress all springs under a car a distance d, and let y be the compression for a force F. Now, as the distortion of perfectly elastic bodies follows Hooke's law of elasticity, viz.: Distortions are proportional to the distorting forces; and as bars, beams, springs, or even structures within the elastic limits follow this law in deflection, compression, extension, etc., so closely that it may be taken for the true law, then we may make the following simple statement for a car on its springs, viz.:

$$P : d :: F : y,$$

or

$$F = \frac{P}{d} y.$$

Applying a simple expression of dynamics for the action of a force on a mass, we have

$$\frac{d^2 y}{d t^2} = f = \frac{\text{acting force}}{\text{mass}} = - \frac{g}{W} \frac{P}{d} y,$$

the sign depending on contrary direction of y and F.

Integrating once, we obtain

$$v^2 = \frac{d\,y_2}{d\,t^2} = -\frac{P}{W}\frac{g}{d}y^2 + C = \frac{P}{W}\frac{g}{d}(h^2 - y^2), \qquad (1)$$

the constant being determined on the supposition that $v = o$ at the beginning of a movement where $y = h$, or where an applied compressive force, F, might be suddenly released, allowing the car body and load to bound upward. When the car has risen to the position of rest $y = o$, giving a high value of v, at $y = -h$, $v = o$ again, and this is the opposite point in the amplitude of vibration where a return movement begins, and so on in repetition. The whole amplitude will then be 2 h.

Solving for t, we obtain, using the negative sign

$$t = \sqrt{\frac{W\ d}{P\ g}} \int \frac{\pm\, d\,y}{\sqrt{h^2\ y^2}} = \sqrt{\frac{W\ d}{P\ g}}\ \cos.^{-1}\frac{y}{h}$$

which expresses the time for the car to move from a compression h of the springs up to a compression y. If we make $y = h$, $t = o$ for a starting point of reckoning time, and this corresponds with the position where $v = o$. For $y = -h$, the opposite limit of amplitude of vibration, we have

$$t = \pi\sqrt{\frac{W\ d}{P\ g}} \qquad (2)$$

and this is the time of a simple vibration. Twice this, or 2 t, will of course be the time of a "complete" or "double" vibration, which is the time occupied in a complete movement forward and back to the same point again.

If $y = +h, -h, +h, -h, +h,$ etc.

$$t = \sqrt{\frac{W\ d}{P\ g}}(o,\ \pi,\ 2\,\pi,\ 3\,\pi,\ 4\,\pi, \quad \text{etc.}$$

which indicates isochronous periodic motion independent of the amplitude, and continued indefinitely, and is a case of repeated "harmonic motion."

As an example of calculated time of vibration of a freight car and load on its springs. the Pennsylvania Railroad make $P = 66\ 000$ pounds for $d = \frac{7}{8}$ inch, while the car body and its load weigh 55 000 pounds $= W$. Introducing these, and we find for the time of a complete vibration of such a car on its springs,

$$t = 0.14 \text{ seconds, and } 2t = 0.28 \text{ seconds,}$$

which is about the same as some of the periods of bridge vibration given in Table No. 5 for loaded bridges.

The time of vibration of a locomotive on its springs may be similarly calculated.

If $W = P$,

$$t = \pi\sqrt{\frac{d}{g}}, = 0.16\ \sqrt{d} \text{ nearly,}$$

if d is expressed in inches. That is, the time of a simple vibration of a weight attached to a spring is equal to 0.16 times the square root of the deflection in inches of that spring due to placing that weight upon it.

If $d = 1$ inch, $2t = 0.16$ second, as for a car body and load which would settle the springs 1 inch.

For a bridge with parallel chords, the exact calculation is more diffi-

cult. The bridge will vibrate similarly as would a rod supported at its ends, the differential equation of the amplitude curve* for which is one of the fourth order. But it is unnecessary to go to this refinement, because the bridge is not precisely like a rod in vibrating. Probably the amplitude curve for a bridge is very nearly circular, differing from it by greater convexity near the abutments where the panel diagonals are most strained, the chord strains being uniform. Regarding it as a circle, an almost exact formula will be obtained. But, for convenience, first compare the vibration of the bridge, where the amplitude curve is regarded as a parabola having a middle ordinate d, with a hypothetical equivalent vibration of the whole bridge from end to end to a uniform amplitude of $\frac{2}{3}d$. In this comparison the relation $\frac{2}{3}$ depends on the well known relative heights of a parabolic section and of a rectangle of equal area.

To show that this comparison is legitimate, reference is had to a principle of harmonic motion, viz.: When a body vibrates through an amplitude $2h$ from the action of a force varying as the distance from the middle point of amplitude, then the acting force at the limit of amplitude is equal to the centrifugal force for the same body whirling in equal time in a circle of diameter $2h$.

The truth of this principle is made evident from the fact that when a stretched string vibrates, as on a violin, the tone resulting is the same whether the string vibrates in a plane, or swings around, describing a conoid of revolution. In either case the force operating to return a given weight of an elementary piece of string in plane vibration, when it reaches the limit of amplitude, is the same as the centrifugal force of like weight similarly situated, at any moment for the conoidal vibration. Hence we may consider the question from the standpoint of centrifugal force.

Thus for the case of the car, the centrifugal force is

$$P = M\frac{v^2}{d} = \frac{W}{g}\,\frac{4\,\pi^2}{t^2}\,d \qquad (3)$$

whence

$$t, = 2\,\pi\sqrt{\frac{W\,d}{P\,g}} = 2\,t \qquad (2)$$

the same expression as previously found for the corresponding complete vibration. Here P varies as d, so that for any point in the amplitude curve, the value of P for an element of mass is proportional to the ordinate for that element. Hence, as the total value of P is to be made the same in the comparison, we have only to find the height of a rectangle whose area and length equals the parabola, and use that height in a formula like the above. Calling d the middle ordinate of the assumed parabolic curve of deflection or amplitude curve of bridge, we may adapt the above formula (2) for car vibration to the case of the bridge with parabolic amplitude curve by putting $\frac{2}{3}d$ for d, viz.:

$$t = \pi\sqrt{\frac{2}{3}\,\frac{W d}{P g}} \qquad (4)$$

where W represents the combined weight of the bridge and uniform

* The expression amplitude curve here means the curve assumed by a vibrating rod when supported at its ends, and freely vibrating between, particularly for the limits of amplitude. This curve resembles that seen by observing a vibrating violin string.

load, while P is the weight of the uniform load which causes the middle deflection, d.

If we regard the amplitude curve as of the form of the segment of an ellipse where $d =$ a quarter of the minor axis, then we should adopt $.7\ d$ in place of $\frac{2}{3} d$, and the actual ordinate probably lies between these.

As an example of calculated time of bridge vibration, take the case of bridge 10 in Table No. 5, where the deflection $= 0.4$ inch, the weight of bridge $= 223\,575$ pounds, and assume the train load at about 1 400 pounds per foot, then

$$2\,t = 0.24 \text{ second,}$$

while the values in the table for like conditions range about at 0.29 second.

For the elliptic value $.7\ d$

$$2\,t = 0.25 \text{ second.}$$

differing inconsiderably from the result 0.24 obtained for the ordinate $\frac{2}{3} d$. The value $.68\ d$ is probably nearer the actual one.

For the case of a concentrated load at mid-span, and weight of bridge neglected, the formula (2) applies without modification, because here the bridge simply serves as the spring.

When a locomotive is at mid-span, followed by a train much lighter in load intensity, we have nearly the case of a concentrated load, a uniform load for a half span, and a uniform load of bridge for whole span. Calling the weights W, W_1 and W_2 respectively, and applying the principle of centrifugal force, we have for the total centrifugal force:

$$P = \frac{W\,4\,\pi^2}{g\,(2\,t)^2}\,d + \frac{W_1\,4\,\pi^2}{g\,(2\,t)^2}\,\frac{2}{3}\,d + \frac{W_2\,4\,\pi^2}{g\,(2\,t)^2}\,\frac{2}{3}\,d.$$

$$= \frac{\pi^2\,d}{t^2\,g}\left(W + \frac{2}{3}\,W_1 + \frac{2}{3}\,W_2\right)$$

whence

$$t = \pi\sqrt{\frac{d}{P\,g}\left(W + \frac{2}{3}\,W_1 + \frac{2}{3}\,W_2\right)} \qquad (5)$$

which is adapted to the case of an engine alone by making $W_1 = 0$. P is here the total weight concerned in causing the deflection d.

As an example, take the seventh case of bridge 10 of Table No. 5, counting the tender in as part of the train, at 1 400 pounds per foot. Then $W = 72\,150$ pounds; $W_1 = 84\,000$ pounds for 60 feet; and $W_2 = 223\,575$ pounds. But $W + W_1 = P$ only are concerned in the observed value of $d = 0.49$ inch.

Introducing these and

$$2\,t = 0.30 \text{ second.}$$

while the observed value given in Table No. 5 is 0.29.

For a bridge and uniform load of cars, the car springs being considered, let $W_2 =$ the weight of the bridge, w the weight of the cars above the springs, w^1 the weight of the trucks, etc., below the springs, and h the compression of the springs due to the car load; then, applying the principle of centrifugal force,

$$P = w + w^1 = \frac{\pi^2}{t^2\,g}\left(w\,\frac{2}{3}\,(d+h) + w^1\,\frac{2}{3}\,d + W_2\,\frac{2}{3}\,d\right)$$

or

$$t = \pi \sqrt{\frac{2\,d}{3\,g}\left(1 + \frac{w\,\frac{h}{d} + W_2}{w + w^1}\right)} \qquad (6)$$

In this formula the greater the value of h the greater is t, so that the time of vibration of bridge and load is increased by the presence of the car springs.

As an example, take the same as employed in testing formula (4), where

$W_2 = 223\,575$ pounds.
$w = 1\,100 \times 148 = 162\,800$ pounds.
$w^1 = 300 \times 148 = 44\,400$.
$h =$ compression for w, and $= .5$ inch.
$d = .4$ inch, the mid-span bridge statical deflection due to $w^1 + w$.
Then

$$t = 0.29 \text{ second.}$$

a value which agrees better with the observed time of vibration noted in Table No. 5 than the result obtained from (4), and it also agrees well with the time of vibration of a car on its springs, as noted in the example for formula (2). These facts are in support of the supposition that when a bridge vibrates the cars also vibrate on their springs.

The close agreement with the observed times of vibration of these results, calculated by the principles of dynamics, is believed sufficient to show that the theory of bridge vibration is not a myth.

These formulas were used in calculating the times of vibration, and vibrations per inch. given in the tables as "calculated;" except that the work was shortened by determining the constants from the observed results; one set of constants being used for the uniform load throughout, and another set for the engine at mid-span.

It may seem that the span and depth of truss should enter the formulas as well as the weight of bridge. Their absence is to be accounted for in the fact that the deflection d will vary with the span and depth. To determine the law of relation of d to the span and depth in similar trusses proportioned for like maximum strains, the elongation of the iron will be the same in all cases, viz., $\frac{5}{1000}$ to $\frac{6}{1000}$ of an inch per foot for a 10 000-pound working strain. By aid of this fact, and an outline diagram of truss, it is easy to see that d varies directly as the square of the span, and inversely as the depth, so that d varies directly as the span in diagramatically similar trusses equally strained.

Causes of Bridge Vibration.

Notwithstanding the fact that a bridge, a car, etc., may vibrate, and that the vibration period may be calculated theoretically, yet we would not expect such vibration to originate without due cause. In studying this we must distinguish between mere lurches of bridge without law, and genuine vibration. The diagrams indicate a considerable tendency to non-systematic lateral movements, much more so than to vertical. This is believed to be due largely to the wandering of the trucks from side to side on the clearance between gauge of rails and wheels, and also to crooks in the rails. The latter may give cause for irregularities in the diagrams of vertical movements, also low or worn joints, flat wheels, etc.

Hence irregularities may appear in the diagrams. But genuine vibration must be regular, at least in vibration periods, and we would never expect an absolutely instantaneous ending of vibratory movement, nor even beginning, unless caused by a blow.

Relative to the operation of the three causes named under "Vibrabration of Elastic Bodies:"

First.—Almost a blow is struck upon the bridge when a locomotive at 40 miles per hour drops its heaviest part upon it within a space of less than half a second. Undoubtedly the effect of this would be modified by bridge camber, but in any case it would seem that the bridge would be depressed somewhat beyond the position of equilibrium, then return, or spring back, etc., or vibrate. The diagrams give evidence of such action to some extent, notably Nos. 137 and 140, Plate VI. But this half-second blow would occupy only an eighth of an inch length of diagram, and as a vibration would occupy less than a tenth of an inch, it would seem that the effect of this blow should be fully developed within a space of a quarter of an inch at the initial point of diagram. Though 137 and 140 show it, the others given on the plate do not so much; 61 and 164 almost none at all. In none do we find an intense vibration established within the first quarter inch; on the other hand, at *C*, in 61, 173, 189 and 192, we find a high intensity of vibration established at about a half inch from the initial point of diagram, which, from the above considerations, must be due to some other cause than the plunge or "pitch" of the engine upon the bridge; particularly so inasmuch as in some diagrams we find the highly developed vibrations, and in others not, as witness the points *C* in 148, 160 and 167.

Second.—Releasing the bridge from strain, either partially or wholly, in no instance has been found sufficient to cause vibration.

Third.—Repeated impulses, mentioned under "Vibration of Elastic Bodies," is found to be a most potent cause of bridge vibration, as amply witnessed by the diagrams. What other cause can be assigned for the intense vibration recorded at *L* on No. 140, also *E* and *G;* and at *E* and *G* of 137, or *D* on 148 and 160; or again, at *E*, *F* and *I* on 164, and at *C* on 173, 182 and 192. A large number of other diagrams taken present like characteristics.

Evidence that these instances of vibration originate in repeated impulses, is found in the fact that the vibration never starts abruptly into full intensity, but, on the other hand, that it usually increases nearly uniformly from comparative quietude to a wide amplitude; a marked instance of this being presented in the band *K L* of 140.

Such repeated impulses are found to arise in connection with the engine, and unavoidably as now built; and also, under certain conditions, in connection with the train, from the following causes, viz.:

First.—Impulses due to the engine find cause in the non-balance of the drivers, there being an impulse downward when the excess of

balance is downward, and upward when the excess is upward. These are sure to occur every time an engine passes over a bridge, but vibration will not occur unless the times of revolution of drivers coincide with the periods of double vibration, in which case the effect is cumulative, and intense vibration is the result.

All the points *C* in diagrams 61, 173, 182, 192 and others, are examples of cumulative vibration due to the non-balance of engine drivers, while points *C* in diagrams 148, 160, 167, etc., are free from vibration owing to want of harmony between revolution and vibration periods.

The above considerations suppose a rigid floor of bridge; but when the floor beams are stiff and the stringers flexible, there will be greater variation in the vibration intensity produced, because when the excess of balance strikes down at mid-panel and up at panel points, the engine will fall and rise through an increased amplitude with corresponding effect on the bridge, while for contrary conditions there will be the opposite effect. To secure the greater impulses at every panel of bridge, the panel length must agree with the drive-wheel circumferences. With all conditions favoring, viz.: flexible stringers, excess of balance down at mid-panel, equality of panel length and driver circumference, and also of revolution and vibration periods, the greatest amplitude at *C* will be developed. This coincidence of conditions, though rarely occurring, is likely to result in serious cumulative vibration.

Second.—Impulses due to the train will occur with some bridges, and with others not, and depend on the circumstances of flexible stringers and coincidence of panel length with half-car length. When these are both satisfied, the speed of the train must be a half car for each complete vibration; then, as each car has two trucks, and as in a freight train the trucks are not far from uniformly distributed, it follows that when one floor beam is under a truck, each and every floor beam will be under a truck or nearly so; and then, with flexible stringers, it follows that as the trucks strike the mid-panel they will drop a little by reason of the yielding stringers, and when they reach the floor beams they will be correspondingly lifted, thus either causing all the cars in the train to fall and rise, or the bridge to rise and fall, or both. Under these circumstances a downward impulse will be imparted to the bridge on each arrival of the trucks at mid-panel; this indeed at each and every panel throughout the length of the bridge. A ten-panel bridge will thus receive ten simultaneous impulses at each complete vibration of the bridge, or twenty impulses for each car length of advance of train, or about eight hundred to one thousand impulses per train with cumulative effect.

But all this presents nothing new in principle. The breaking of step of marching soldiers when crossing a bridge is for the purpose of avoiding cumulative vibration. The famed fiddler might break the bridge if the jerks of his arm "keep time" with the vibrating bridge.

In the car and its load we may find conditions favorable or unfavorable for vibration of bridge. If the load is solid, like coal, pig iron, etc., it will offer less resistance internally to vibratory influence than if it be yielding, like bales of hay, live stock, etc.

Inasmuch as the car and load are found to vibrate on the car springs in about the same time as an ordinary Pratt truss of 150 feet span, there would be harmony between them. Then the impulses, acting through a given amplitude, such as the versed sine of spring of stringers, would occasion a given vibratory amplitude of bridge; and for the cars a greater one when on springs than when not. Then this increased car amplitude would excite an increased reaction upon bridge at limits of amplitude, and hence in turn an increased amplitude of bridge.

In the bridge itself we find conditions which act to modify vibration; for instance, as the chords lengthen and shorten in response to the vibratory strains, a resistance to this like sliding of chord terminals or pedestal blocks will hinder vibration.

The position of pedestals and form of truss also have a bearing on vibration. A fish-shaped truss, with both chords joined into common terminal blocks situated at the neutral axis of truss, would vibrate without disturbing those blocks. Wood stringers extending from the bridge out into the bank, and at such height in the bridge as to necessarily slip when the bridge vibrates, will hinder the vibration.

From these considerations of the three causes of bridge vibration, it appears that the first occasions but mild results, scarcely worth considering; the second, none; while the third is capable of producing results of unknown severity, and which may be styled cumulative vibration in bridges.

Cumulative Vibration in Bridges.

The conditions favoring cumulative vibration in railroad bridges may be classified as follows, viz.:

Primary.

Under suitable train speed:

First.—Non-balance of drive-wheels in locomotives as now constructed.

Second.—Yielding stringers in bridge floors, with equality of panel and half-car length.

Secondary:

a. Vertical vibration of car on its springs.

b. Equality of drive-wheel circumference and panel length of bridge.

c. Excess of non-balance of drivers down at mid-panel.

d. Free pedestal blocks, as on expansion rollers.

e. Fish-shaped trusses, pedestals at neutral line.
f. Absence of parts overreaching banks causing friction.
g. Firm instead of yielding load.

Evidence of the Operation of the Cause and Favoring Conditions Producing Cumulative Vibration, as Found in the Present Indicator Work.

First.—As due to an engine heading a train.

In the following table are given the results obtained from nine diagrams of passenger trains, the first three being illustrated in Plate V. These three exhibit unusual sinuosity at *C*, while other portions of diagram are nearly smooth. These sinuosities *C* were recorded as the locomotive was going over the bridge.

TABLE No. 6.

Synchronism of conditions favoring cumulative vibration for case of an engine with a train following over a bridge.

Diagram.	Per Ct.	Obs. Vib.	Cal. Vib.	Rev.	Floor Beams.
173	22.6	13.3	13.6	13.5	15:8
182	21.7	15.0	13.8	16.2	18.9
192	22.1	12.8	14.0	14.0	16.3
129	18.2	13.0	13.3	11.0	12.1
134	16.2	13.3	13.0	11.5	12.6
141	17.8	13.5	13.0	....	13.2
147	14.3	13.3	13.0	12.8	14.4
162	14.1	12.6	13.4	14.0	....
171	16.4	13.5	13.0	14.0	....

This table is made up mostly from Table No. 2, selected with reference to the high percentages of superadded deflection due to vibration, as given in column 2; the third column is the number of complete vibrations per inch of diagram as observed; the fourth column is the same calculated; the fifth column is the number of revolutions of drivers per inch of diagram; and the sixth column is the number of floor beams passed per inch of diagram.

According to the first primary condition above, the fourth and fifth columns should agree, as in fact they do very nearly. A comparison of the fifth and sixth columns shows that the secondary condition *b* is very nearly realized, so that we find reason to expect the high percentages actually recorded.

Where there is a want of harmony in the quantities represented in the last three columns, the percentage of added deflection due to vibration

will generally be much less, as shown in the following table, the first five diagrams of which are illustrated on the plates.

TABLE No. 7

Discord of conditions for cumulative vibration for case of an engine with a train following over a bridge.

Diagram.	Per Ct.	Obs. Vib.	Cal. Vib.	Rev.	Floor Beams.
137	6.4	....	12.7	11.9	9.5
140	15.4	12.3	12.7	12.4	9.8
160	5.0	....	13.8	12.4	11.3
164	9.0	....	12.7	11.1	9.7
167	2.5	....	12.5	9 to 11	8.8
131	2.6	....	12.6	7.8	6.5
157	2.2	....	12.4	10.6	8.4
158	6.1	....	13.4	10.0	8.4

The last three columns are seen to be much more discordant than in the like columns of the preceding table, with a corresponding lower percentage of superadded deflection, except in 140, where the vibration was sufficiently well defined to be read off, and which is seen to harmonize very well with two of the last three columns. 164 gives a percentage neither low nor high, and for this we find fair agreement in the last three columns. The last four comparisons of the table show greatest discord, and, with the exception of the last, the lowest percentages.

There are some exceptions to the rule, with unaccountable causes. An explanation may yet be found for the exceptions when more elaborate indicators and methods shall have been put to the task. The problem involves a large amount of complexity, the load on the tender being one variable element.

The first of the two tables, Nos. 6 and 7, happens to represent passenger trains only, and the second, freight; but it is difficult to see why the difference in the train should make a difference in the vibration as due to the engine itself, except for the fact of the usually higher speed and larger drivers of passenger trains. The one exception in Table No. 7 to the general low percentage shows that we may have high percentages for an engine followed by a freight train; and on the other hand diagrams 30, 39, 65, and about a dozen others not given in Table No. 2, from passenger trains show small percentages, so that both high and low percentages have been observed for the engine followed by both passenger and freight trains. Hence the dependence of the vibration upon favoring conditions, and their independence of the kind of train.

Tables Nos. 6 and 7 show a better agreement of driver revolutions

than of floor-beam frequency with the calculated time of vibration; and in Table No. 3 we find higher percentages for rigid than for yielding flooring; facts which favor non-balance of engine-drivers as a disturbing cause rather than floor-beam frequence, and rigid rather than yielding stringers for the case of an engine at the head of a train.

But we observe that there is not exact agreement of drive-wheel circumference with panel length, and that this throws the excess of balance "out of time" as the wheels repeat their circumferences along the bridge, thus rapidly cutting down the combined influence of the stated conditions "2 *d*" and "*c*." But when there is agreement, it is undoubtedly the fact that in at least that half of the cases when the excess of non-balance arrives upon the center of panel in the down position, the flexible stringers will conspire with the drivers towards a percentage largely in excess of that obtainable on rigid flooring.

Second.—Vibration due to train.

Relative to cumulative vibration as due to the train and not the engine, we see by referring to the plates that the spells of vibration are of much greater duration. The engine may pass over the bridge in three seconds, whereas the recorded belt *D H*, in 137, 2.5 inches long, was about ten seconds in making, and a portion of train two and a half times the length of the bridge passed over during that time. 140 and 164 present belts of about two inches in length; hence strong vibratory action is transferable from one set of cars to another along a train. From this fact the possibilities of cumulative effect from long trains are seen to be very great, not only as regards duration, but acquired intensity.

But the observations show that the conditions of loading vary greatly along the train, so that a vibratory condition may be destroyed by the passage of one or two cars differently loaded.

A change of load intensity is sure to break up the continuity of vibration, but there are other influences less apparent which come to act. On 137 the loading appears to be nearly constant to *L*, and almost precisely so to *K*. Judging from the diagram the vibration should continue at least to *K*, and, in the absence of information, the cessation of vibration at *H* would necessarily be unaccountable. As previously stated, numerous unexplainable facts have appeared which might be made clear when more elaborate indicators and methods shall be put in use. The truth of this cannot be more forcibly impressed than by here pointing out the real cause of the lost intensity at *H*, which, fortunately, in this case we know. At *D* there followed three cars of 33 feet length. Then came fourteen box cars of 28 feet length, followed by stock cars of 33 feet length. The whole number of cars in the train is thirty, of which the fourteen box cars of 28 feet length is nearly half (allowing, of course, for the engine). Hence for the particular speed of this train the 28-foot cars were of favorable length and load, while

the 33-foot ones were not. Another fact worthy of note here is with regard to the loading itself, the cars following *H* being stock cars, while those between *D* and *H* were not. Thus the condition *g*, of rigid loading, finds support, while all doubtless can appreciate the antipathy of a cow's back for elastic vibration.

Hence the change in car length and condition of car load at *H* constitute two reasons why the vibration should cease at *H*. Similarly the neck in the 2-inch belt of 140 might be explained provided the necessary information were at hand. But reliable specific notes concerning the trains were hard to get, for the reason that the observer's opportunity for it lasted but a few seconds—for 137 about twenty-four seconds.

The bands of vibration record on the plates are seen to begin by a rising scale, showing that the origin of the vibration is not a sudden shock, but a succession of impulses, as already explained. Such impulses were not anticipated as occurring in connection with the train itself; and the indicator greatly surprised us when it brought out the records, the eyes of my field observer, Mr. E. O. Ackerman, being the first to behold them. The record once an existing fact, an explanation was soon found in the yielding elastic stringers, and equality of half-car length and bridge panel.

The various results of this cumulative action are collected in Table No. 4, the greater part of which were obtained from the one bridge, 11, for which the panel length is 15 feet 8 inches, or very nearly the half-car length, and the stringers were wood. Bridge 10, on the same road, has panels of about the same length, but with stiff iron stringers.

An anomaly is found in diagram 61, in the fact of apparent cumulative vibration for train, when no explanation can be found for it in the field notes. Of the twenty-three diagrams taken from the same bridge, however, this is the only one exhibiting such record. The bridge panels are 8 feet, and the rails are laid directly on the floor beams, placed 2 feet or less between centers.

In the seven diagrams of bridge 7, no cumulative results were observed, as indeed might be expected from the discordant panel length of 19¼ feet.

For all entries in Table No. 4, except the one diagram, 61, we have a near coincidence of half-car and panel length. Hence, column 14 might be regarded as nearly representing another like it, headed "floor beams per inch of diagram."

Now, columns 12, 13, and 14, or virtually four columns, are seen to agree almost perfectly in conditions favoring cumulative vibration, whereas Table No. 2, so far as it presents the figures for other diagrams for the train, does not exhibit like agreement in the columns referred to. See 126, 132, 144, 145, 157 and 189.

Other bridges upon which the indicator was placed, with panels differing much from 15 feet, gave no cumulative records for trains.

For conditions "*d*" and "*f*," the experiments as a whole showed that bridge 11 was much more sensitive to vibration than the other bridges "indicated." This bridge had square abutments, with seats for the pedestal blocks up free and clear, with expansion rollers at one end, and all apparently in condition favoring vibration as regards "*d*" and "*f*."

The Lateral Vibration.

The lateral vibration for engine and train appears to be much more accidental in its character than the vertical vibration. When the engine strikes the bridge, the latter makes a few lurches apparently without law, but, afterwards, while the train is passing, it seems to settle down by spells to approximate regularity. Thus, at *O*, in 137, the sinuosities are quite regular, but fade away, until approaching *P* fresh lurches occur from some cause. No specific explanation can as yet be given for such freaks of the indicator pencil. It would seem that the cause is connected with that forming the indentation at *F*, as though a car here passed the bridge with loading one-sided. But this is only conjectural. See also first page under "Causes of Bridge Vibration."

The times of lateral vibration, and number per inch, given in the tables, have been made out from such portions of *M N* as are most systematic. These seem to run approximately at about twice the period, or half as many per inch as for the vertical vibration. But this, in the bridges examined, is evidently accidental, as the period of vibration depends largely on the stiffness of the trussing, it being less where the lateral truss-rods begin at end of bridge with 2-inch bars instead of 1.25 inch. These rods for bridge 11 were 1¾ inches in diameter at the abutment panels.

The amplitude of the lateral vibrations as given by the diagrams is to be found in Table No. 2. As soon as the deflection and strain due to the wind pressure is known, the superadded strain due to the lateral vibration can be approximately calculated. But the diagrams give nothing from which to make out a percentage between vibration and wind strain.

Frequency and Dynamic Effect of Observed Cumulative Vibration.

Of the 193 train transits indicated, the 25 of Table No. 3 give an average percentage of 18.4 for the engine heading a train, which is about one in eight.

For the train itself, Table No 4 gives an average maximum percentage of 26.4 for the eleven transits noted, which is 1 in 18. For both it is about one in five. That is, according to observation, cumulative vibration occurs as often as once in every fifth time a train goes over certain bridges.

TABLE No. 1.

Giving Particulars of Railway Bridges to which the Indicator was Applied in 1884.

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Number in order occupied.	te	oad.	Designation by Railway Company.	Location.	Kind of Bridge.	Span. Ft. In.	Number of Panels.	Length of Panel. Ft. In.	Depth of Truss. Ft. In.	Floor Beams.	Stringers.	Height of Track above Chord Pins.	Number of Diagrams taken.	Remarks.
1	Aug. ..	Little Miami.	....	Columbus, O. Over the Scioto Riv.	Pratt Truss.	136 0	9	15 1⅓	About 24′	28″ deep and strong.	Iron I sec. 18″ deep, and flanges 10½″ across.	Abt 24″	14	
2	Aug. 19 and 20.	N. Y. P. & O. Main Line.	70	About 2½ miles N. E. of Dayton, O. Over Mad River.	2 Spans through Pratt Truss.	140 6	9	15 7⅓	24 0	Skewed ½ panel 31″ deep, with flanges 10½″ across.	Iron I sec., 24″ deep, flanges 10½″ across.	About level with top of lower chord.	9	Bridges continuous over middle pier. Lower chords of panel next to mid-pier are compression members.
3	Aug. 21 to 23.	do	69	About 6 miles N. E. of Dayton, O.	do	156 0	10	15 7⅕	24 0				16	
4	Oct. 16	Penn. R. R. Del. Exten. of Phila. Div.	Arsenal Bridge.	Over Schuylkill River.	Through Pratt Double Intersec.	189 7	20	9 5¾	19 0	Floor beams directly on lower chord 4 per panel 8″ by 12″	Stringers under rails; no ties.	Abt 24″	4	Has b'n condemned for two years.
5	Oct. 17.	Penn. R. R. Phil. & West. Chr. Div.	1, or Mayland B'g	About ½ mile W. of So. St. Sta., Phila., Pa. Over Mayl'd Creek	Two separate Deck Pratt Trusses for double track. North truss.	128 3	9	14 3	16 6	Floor beams are from 18″ to 2′ from center to center; they are 8 by 14 in., and rest directly on lower chord	No stringers. Rails are laid directly on the floor beams.	21″	12	Guard rails are 8 by 8 inches.
6	do	do	do	do	South truss.	128 0	16	8 0	15 0			21″	11	
7	Oct. 18.	Phil. Wlm & Bal. R. R.	Chester Bridge.	600 or 800 ft. W of Chester. Over Chester Creek.	Through Pratt Double Track.	154 6	8	19 3¾	26 0	3′ 8″ dp,; suspended.	Iron I sec., 22″ deep, 6′ apart c. to c.	On level with pins.	7	Very strong and stiff bridge.
8	Oct. 23.	B. & O. R. R. Main Line.	Harper's Ferry Bridge.	Over the Potomac River and Canal.	7 spans through Bollman 4th span occp'd.	135 0	10	2 ends=17½′ Others=12½′	17 6	16 by 12 in. boxed.	2–15″ I beams, 5″ across the flanges.		4	For rail and wagon road, and has 3 trusses.
9	Nov. 21 and 22.	N. Y. P. & O. Main Line.	55	Just S. W. of Greencamp, O. Over the Big Scioto.	Through Pratt Truss	150 0	10	15 0	24 0	Riveted to columns 36″ dp.; flanges 10½″ across.	I sec., 26″ dp.: flanges 9¼″ across.	4′	7	
10	Nov. 24 to Dec. 24.	do	57	Between Claibourne and Broadw'y, O. Over Beans Creek.	do	148 0	10	14 9⅗	24 0	do	do	4′	32	
11	Nov. 29 to Dec. 15.	do	32	Abt. 2½ miles W. of Leavittsburg, O. Over Mahoning River.	do	141 0	9	15 8	About 24′	Suspended iron I sec., about 30″ deep.	3—8″ by 18″ wood; resting on floor beams.	1′	60	
12	Dec. 1.	N. Y. P. & O. Mahoning Div.	9	do	do	143 0	8	17 10½	24 0	Suspended iron I sec.	Iron I sec., 16″ deep; riveted to floor beams.	On level with pins.	4	
13	Dec. 2 and 3.	N. Y. P. & O. Main Line.	29	About 1 mile E. of Orangeville, Pa.	Through Pratt Double Intersec.	156 6	11	14 2¾	About 24′	Suspended iron I sec.	Wood 16″ dp.: resting on floor beams.	16″	13	

Total number of bridges, 13.

Total number of diagrams, 193.

TABLE No. 2.

General Table, including Results from all Diagrams of Special Interest.

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Serial number. 0	Bridge (see Table No. 1.)	Number of Diagram. 1	Train. Passenger or freight. 2	Number of cars. 3	Number of cars from Columns 12 and 15. 4	Engine. Class and length, pilot to buffer. 5	Engine. Diameter of drivers. 6	Speed. Miles per hour.* 7	Abscissa, or distance from initial point of diagram. 8	Ordinate, or statical deflection of bridge.‡ 9	Amplitude of vibration.‡ 10	Percentage of statical deflection superadded by vibration. 11	Number of vibrations per inch of diagram.† Observed. 12	Number of vibrations per inch of diagram.† Calculated. 13	Number of half-car lengths per inch of diagram. 14	Inches of diagram per train. 15	Number of revolutions of engine drivers per inch of diagram. 16	Lateral movements. Number of vibrations per inch of diagram.† 17	Lateral movements. Amplitude.‡ 18	Remarks on movements of bridge. Vertical. 19	Remarks on movements of bridge. Lateral. 20
							Inches.		Inches.	Inches.	Inches.	Per cent									
18	2	4	E. B. Pass	5	....	Pass. Eng..	66	....	0.32	0.70	0.40	28.6	14.5	....	....	1.4	....	....	0.25	Cumulative.	
20	2	6	W. B. "	7 or 8	....	" 48'	66	46.5	0.30	0.60	0.28	23.3	15.5	....	....	....	16.2	....	0.15	"	
21	2	7	E. B. "	6	....	" 48'	66	37.9	0.40	0.72	0.35	17.3	10.	....	....	....	13.2	6.	0.3	"	
22	2	8	W. B. L. Ft......	10 box	10 box	Frt. Eng..........	..	28.8	0.60	0.72	0.14	9.7	....	....	12.5	....	....	....			
..	..	..	....	and	and	....	..	....	1.00	0.54	0.30	29.8	13.	....	....	....	....	....	0.10	"	Irregular.
..	..	..	....	1 coach.	1 coach.	....	..	....	1.50	0.40	0.10	12.5	....	....	....	....	....	....			
23	2	9	W. B. Pass.......	....	....	Pass. Eng.........	66	57.0	0.30	0.68	0.30	22.0	19.	....	....	....	19.8	....	....	"	
27	3	4	W. B. "	4	....	"	66	46.6	0.30	0.73	0.26	17.8	16.	....	....	....	16.3	....	0.80	"	
29	3	6	W. B. "	6	....	" 48'	66	47.6	0.27	0.66	0.28	21.2	17.	....	....	....	16.6	....	0.50	"	
30	3	7	E. B. "	4	....	" 48'	66	42.7	0.35	0.76	0.30	19.8	14.	....	....	....	14.9	....		"	
31	3	8	W. B. L. Ft.....	..	....	Eng. No. 92	..	Low.	1.40	0.40	0.15	18.7	16.	....	....	....	....	....	....	Slightly cumulative.	
..	..	..	....	..	....	....	..		....	0.00	....	....	20.	....	....	....	....	....	....		
32	3	9	W. B. Pass......	4	....	Pass. Eng.........	66	46.2	0.30	0.81	0.30	18.5	16.	....	....	....	15.4	....	....	Cumulative.	
39	3	16	W. B. "	4	....	"	66	48.7	0.26	0.71	0.14	9.6	17.4	....	....	....	17.0	....	. . .		
44	5	1	W. B. "	2	....	Eng. No. 202.......	..	....	....	0.94	0.30	15.0	....	....	....	0.1	....	....	0.44	"	Not smooth.
48	5	5	W. B. "	4	....	" 202......	..	....	0.35	0.88	0.28	15.9	....	....	....	0.7	....	....	0.30	"	
49	5	6	W. B. "	1	....	" 216......	..	....	0.30	0.94	0.26	13.8	....	....	....	0.2	....	8.	0.45	"	
61	6	6	E. B. Ft........	8	8.9	" 216.....	..	Abt. 40.	0.40	1.40	0.32	11.4	13.5	....	13.5	1.4	....	No latl.	....	"	
..	..	..	....	..	....	....	..	....	0.65	0.80	0.30	18.7	17.	....	for box	....	....	....	....	"	
..	..	..	....	..	....	....	..	....	1.20	0.74	0.06	...	....	....	cars.	....	....	....	....	"	
65	6	10	E. B. Pass......	3	....	Eng. No. 213......	..	....	0.40	1.35	0.26	9.6	13.	....	....	0.9	....	....	....		
85	10	1	W. B "	5	....	Pass. Eng.........	..	....	0.37	0.78	0.26	16.7	16.	....	....	1.0	....	....	....	"	
89	10	5	W. B	5	....	"	..	....	0.38	0.78	0.30	19.2	15.2	....	....	1.1	....	....	....	"	
96	11	2	W. B. Frt.......	..	....	Condn. 50'	48	....	....	1.00	0.38	19.0	. . .	....	....	....	....	....	....	"	
103	11	9	W. B. "	..	....	....	..	....	. . .	0.72	0.20	13.9	....	....	....	....	....	....	....	slightly.	
124	13	12	W. B. Pass.....	..	....	Std. H. P. 48'	66	....	0.55	0.83	0.38	23.4	....	....	....	....	....	....	....	"	
126	11	1	E. B. Ft........	33	....	Ft. Eng...........	..	21.1	0.52	0.96	0.12	6.5	....	....	....	7.8	....	....	0.30	Sinuosities irregular.	
..	11	..	....	..	....	....	..	....	2.52	0.67	0.10	7.4	11.0	11.0	8.8	....	....	....	0.20	" "	
129	11	4	E. B. Pass......	10	....	Std. H. P. 48'	66	33.1	0.37	0.82	0.30	18.2	Abt. 13.	13.3	....	2.9	11.0	....	0.40	Cumulative.	
..	11	..	....	..	....	....	..	....	1.80	0.47	0.05	5.3	....	....	....	....	....	....	....		
..	11	..	....	..	....	....	..	....	....	....	....	...	....	....	....	....	....	6.2	....		
130	11	5	E. B. Ft........	32	....	Ft. Eng..........	..	23.4	0.55	0.95	0.10	5.3	....	....	....	7.0	....	....	0.20	....	Sin. irregular.
..	11	..	....	..	....	....	..	....	3.67	0.70	0.14	9.3	11.9	10.9	9.5	....	....	....	0.15	....	"
..	11	..	....	..	....	....	..	....	7.82	....	....	...	....	....	....	....	....	....	....		
131	11	6	E. B. Ft........	30	....	Condn. 50'	48	17.8	0.80	0.97	0.05	2.6	....	12.6	....	8.7	7.8	....	0.25	Sinuosities irregular.	"
..	11	..	....	..	....	....	..	....	3.25	0.75	0.07	5.0	....	....	....	....	....	....	0.06	Slightly cumulative.	"
..	11	..	....	..	....	....	..	....	3.60	0.67	0.16	12.0	12.6	11.0	7.3	....	....	....	0.18	" "	"
..	11	..	....	..	....	....	..	....	3.95	0.70	0.05	3.6	....	....	....	....	....	....	0.15	" "	"
..	11	..	....	..	....	....	..	....	9.75	....	Small.	...	22.0	22.0	....	....	....	....	....	....	"
132	11	7	W. B. Ft........	30	....	Ft. Eng..........	..	21.6	0.60	1.05	0.22	10.7	....	....	8.7	7.2	....	....	0.30	Cumulative.	"
..	11	..	....	..	....	....	..	....	2.80	0.63	0.10	7.8	11.8	11.2	....	....	....	....	0.15	Sin. broken into on	"
..	11	..	....	..	....	....	..	....	5.88	0.90	0.15	8.3	9.5	9.8	....	....	....	....	0.25	both sides.	
133	11	8	W. B. Ft........	11	....	Ft. Eng..........	..	23.2	0.59	1.17	0.07	3.1	....	....	....	2.8	....	....	0.25		
..	11	..	....	..	....	....	..	....	2.25	0.70	0.21	15.0	10.0	10.9	8.8	....	....	....	....	Sinuosities regular.	
134	11	9	W. B. Pass......	4	....	Std. H. P. 49' 7"..	68	34.5	0.35	0.89	0.29	16.2	13.3	13.0	....	1.2	11.5	Abt. 4.	0.45	Cumulative.	Sin. nearly reg.
136	11	11	W. B. Ft........	32	....	Ft. Eng...........	..	23.3	0.62	1.06	0.17	8.2	Abt. 9.7	....	....	7.1	....	....	0.25		
..	11	..	....	..	....	....	..	....	4.52	0.75	0.16	10,8	10.0	10.6	9.5	....	....	....	0.10		
137	11	12	E. B. Ft........	30	32.	Condn. 50'	48	26.1	0.50	0.97	0.12	6.4	....	12.7	....	6.1	11.9	5.7	0.33	....	Sin. irregular.
..	11	..	....	..	....	....	..	....	1.12	0.52	0.13	13.0	....	12.1	....	....	....	5.5	0.20	Highly cumulative, be-	
..	11	..	....	..	....	....	..	....	2.30	0.72	0.46	33.2	11.0	10.7	10.7	....	....	....	0.15	gins at end of 3 33' cars.	
..	11	..	....	..	....	....	..	....	2.90	0.65	0.25	19.2	11.0	11.2	for box.	....	....	4.5	0.30	Where 14 28' box cars be-	"
..	11	..	....	..	....	....	..	....	3.35	0.70	0.42	30.4	11.0	10.9	....	....	....	....	0.10	gin and breaks up at 2	
..	11	..	....	..	....	....	..	....	3.88	0.70	0.02	1.8	....	10.9	9.1	....	....	....	0.18	33' stock cars.	
..	11	..	....	..	....	....	..	. . .	6.77	....	Small.	....	22.0	22.0	for	....	....	....	....		
139	11	14	E B. Ft........	28	....	Ft. Eng..........	..	24.0	0.60	1.02	0.10	4.9	....	....	stock.	6.8	....	....	....	Sin. regularly broken.	"
..	11	..	....	..	....	....	..	....	1.45	0.75	0.15	10.0	9.5	10.6	8.7	....	....	....	0.20	....	"
..	11	..	....	..	....	....	..	....	4.90	0.67	0.02	1.8	....	....	....	....	....	4.5	0.20	....	"
140	11	15	E. B. Ft........	31	31.	Condn. 50'	48	26.6	0.52	0.97	0.30	15.4	12.3	12.7	....	5.9	12.4	....	0.35	....	"
..	11	..	....	..	....	....	..	....	1.00	0.37	0.04	4.9	....	....	....	....	....	....	0.10	....	"
..	11	..	....	..	....	....	..	....	1.62	0.71	0.26	18.4	11.0	10.8	10.9	....	....	....	0.12	Cumulative.	"
..	11	..	....	..	....	....	..	....	2.10	0.57	0.06	5.3	....	....	....	....	....	....	0.18	....	"
..	11	..	....	..	....	....	..	....	2.55	0.62	0.27	22.0	11.0	11.3	10.9	....	....	....	0.22	Cumulative.	"
..	11	..	....	..	....	....	..	....	3.65	0.53	0.05	4.6	....	....	....	....	....	....	0.10	....	"
..	11	..	....	..	....	....	..	....	4.78	0.66	0.07	5.7	....	....	....	....	....	....	0.25	"	"
..	11	..	....	..	....	....	..	....	5.68	0.75	0.46	30.8	10.7	10.6	10.9	....	....	....	0.10	....	"
..	11	..	....	..	....	....	..	....	6.25	0.65	0.12	9.6	....	....	....	....	....	....	....	Cumulative.	"
141	11	16	E. B. Pass......	4	....	Pass. Eng.........	..	35.9	0.72	0.87	0.31	17.8	13.5	13.0	...	1.2	....	....	0.30	"	"
..	11	..	....	..	....	....	..	....	0.90	0.45	0.02	....	....	....	....	....	....	....	....	Sinuosities irregular.	"
144	11	19	W. B Ft.........	6	....	8-wheel Ft........	57	....	0.49	0.84	0.07	....	....	....	....	1.4	....	....	0.15	" "	"
..	11	..	....	..	....	49' 4" "	..	....	1.33	0.32	0.07	....	Abt. 15.	14.1	10.6	....	....	....	0.15	....	"
145	11	20	W. B. Ft........	32	....	Ft. Eng...........	..	....	0.45	1.07	0.20	9.3	....	....	....	6.0	....	....	0.25	Sin. irregularly broken.	"
..	11	..	....	..	....	....	..	....	1.80	0.40	0.07	....	Abt. 11.6	13.2	10.2	....	....	5.3	0.12	" "	"
..	11	..	....	..	....	....	..	....	4.80	0.55	0.15	....	" 11.5	11.9	10.2	....	....	4.7	0.18	" "	"
147	11	22	E. B. Pass......	4	....	Std. H. P. 49' 7"..	68	39.5	0.30	0.87	0.25	14.3	" 13.3	13.0	....	1.1	12.8	....	0.30	" "	"
148	11	23	E. B. Ft........	10	10½	" 49' 4"..	57	....	0.50	0.75	0.02	....	....	....	....	2.2	....	....	0.20	....	"
..	11	..	....	..		....	..	....	0.87	0.50	0.06	....	....	....	....	....	....	5.7	0.20	Cumulative.	Sin. regular.
..	11	..	....	..	mns 16.	....	..	....	1.75	0.52	0.25	23.8	11.1	12.1	10.6	....	....	....	0.10	"	"
..	11	..	....	..		....	..	....	2.35	0.30	0.02	....	....	....	....	....	....	....	0.10	"	"
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Sin. irregularly broken.	"

..	11	..	....	..	....	....	..	....	2.90	0.65	[illegible]	[illegible]	[illegible]	11.2	for box.	....	....	[illegible]	[illegible]	[illegible]	
..	11	..	....	..	....	....	..	....	3.35	0.70	0.42	30.4	11.0	10.9	....	....	....	....	0.10	gin and breaks up at 2	
..	11	..	....	..	....	....	..	....	3.88	0.70	0.02	1.8	....	10.9	9.1	....	....	....	0.18	33' stock cars.	
..	11	..	....	..	....	....	..	....	6.77	....	Small.	....	22.0	22.0	for	....	....	....	....		
139	11	14	E B. Ft.	28	....	Ft. Eng.	..	24.0	0.60	1.02	0.10	4.9	....	....	stock.	6.8	....	....	....	Sin. regularly broken.	"
..	11	..	....	..	....	....	..	....	1.45	0.75	0.15	10.0	9.5	10.6	8.7	....	....	....	0.20	....	"
..	11	..	....	..	....	....	..	....	4.90	0.67	0.02	1.8	....	....	....	....	....	4.5	0.20	....	"
140	11	15	E. B. Ft.	31	31.	Condn. 50'	48	26.6	0.52	0.97	0.30	15.4	12.3	12.7	....	5.9	12.4	....	0.35	....	"
..	11	..	....	..	....	....	..	....	1.00	0.37	0.04	4.9	....	....	....	....	....	....	0.10	....	"
..	11	..	....	..	....	....	..	....	1.62	0.71	0.26	18.4	11.0	10.8	10.9	....	....	....	0.12	Cumulative.	"
..	11	..	....	..	....	....	..	....	2.10	0.57	0.06	5.3	....	....	....	....	....	....	0.18	....	"
..	11	..	....	..	....	....	..	....	2.55	0.62	0.27	22.0	11.0	11.3	10.9	....	....	....	0.22	Cumulative.	"
..	11	..	....	..	....	....	..	....	3.65	0.53	0.05	4.6	....	....	....	....	....	....	0.10	....	"
..	11	..	....	..	....	....	..	....	4.78	0.66	0.07	5.7	....	....	....	....	....	....	0.25	"	"
..	11	..	....	..	....	....	..	....	5.68	0.75	0.46	30.8	10.7	10.6	10.9	....	....	....	0.10	....	"
..	11	..	....	..	....	....	..	....	6.25	0.65	0.12	9.6	....	....	....	....	....	....	....	Cumulative.	"
141	11	16	E. B. Pass.	4	....	Pass. Eng.	..	35.9	0.72	0.87	0.31	17.8	13.5	13.0	...	1.2	....	....	0.30	"	"
..	11	..	....	..	....	....	..	....	0.90	0.45	0.02	....	....	....	....	....	....	....	....	Sinuosities irregular.	"
144	11	19	W. B Ft.	6	....	8-wheel Ft.	57	....	0.49	0.84	0.07	....	....	....	....	1.4	....	....	0.15	" "	"
..	11	..	....	..	....	49' 4" "	..	....	1.33	0.32	0.07	....	Abt. 15.	14.1	10.6	....	....	....	0.15	....	"
145	11	20	W. B. Ft.	32	....	Ft. Eng.	..	....	0.45	1.07	0.20	9.3	....	....	....	6.0	....	....	0.25	Sin. irregularly broken.	"
..	11	..	....	..	....	....	..	....	1.80	0.40	0.07	....	Abt. 11.6	13.2	10.2	....	....	5.3	0.12	" "	"
..	11	..	....	..	....	....	..	....	4.80	0.55	0.15	....	" 11.5	11.9	10.2	....	....	4.7	0.18	" "	"
147	11	22	E. B. Pass.	4	....	Std. H. P. 49' 7"	68	39.5	0.30	0.87	0.25	14.3	" 13.3	13.0	....	1.1	12.8	....	0.30	" "	"
148	11	23	E. B. Ft.	10	10½	" 49' 4"	57	....	0.50	0.75	0.02	....	....	....	....	2.2	....	....	0.20	....	"
..	11	..	....	..		....	..	....	0.87	0.50	0.06	....	....	....	....	....	....	5.7	0.20	Cumulative.	Sin. regular.
..	11	..	....	..		....	..	....	1.75	0.52	0.25	23.8	11.1	12.1	10.6	....	....	....	0.10	"	"
..	11	..	....	..		....	..	....	2.35	0.30	0.02	....	....	....	....	....	....	....	0.10	"	"
149	11	24	W. B. Pass.	3		Pass. Eng.	..	....	0.42	0.75	0.16	10.8	12.3	13.5	....	1.2	....	....	0.15	Sin. irregularly broken.	"
..	11	..	....	..		....	..	....	1.84	0.00	Small.	....	22.0	22.0	....	....	....	....	....		
150	11	25	W. B. Ft.	Abt. 35	(See columns 6, 15, and 16.)	Ft. Eng.	..	....	0.60	1.15	0.17	7.6	11.0	12.5	....	6.7	11.8	....	0.30	" "	
..	11	..	....	..		....	..	....	4.48	0.52	0.15	14.3	10.2	12.1	Abt.10.8	....	....	....	0.10		
151	11	26	E. B. Pass.	9		Std. H. P. 48'	66	29.8	0.40	0.75	0.25	16.7	12.0	13.6	....	2.9	10.0	4.0	0.42	Cumulative.	"
155	11	30	W. B. "	4	4.	" 49' 7"	68	....	0.37	0.87	0.25	14.2	13.5	13.0	....	1.3	11.0	....	0.25	"	"
157	11	32	W. B. Ft.	34	36.	Condn. 50'	48	23.0	0.67	1.12	0.05	2.2	....	12.4	9.4	7.6	10.6	....	0.25		
..	11	..	....	..	....	....	..	....	1.15	0.82	0.20	12.1	10.0	10.4	9.4	....	....	6.0	0.16	Slightly cumulative.	"
..	11	..	....	..	....	....	..	....	1.57	0.52	0.05	4.8	....	12.1	9.4	....	....	....	0.20		
..	11	..	....	..	....	....	..	....	2.50	0.75	0.15	10.0	....	....	....	....	....	....	0.15	"	
..	11	..	....	..	....	....	..	....	2.90	0.55	0.15	13.6	....	....	....	....	....	....	0.20		
..	11	..	....	..	....	....	..	....	3.45	0.72	0.07	5.2	Abt. 8.7	10.7	9.4	....	....	5.5	0.12	....	"
..	11	..	....	..	....	....	..	....	4.00	0.67	0.20	14.7	....	11.0	9.4	....	....	....	0.15		
..	11	..	....	..	....	....	..	....	4.30	0.47	0.05	5.3	....	....	....	....	....	....	0.10	Bridge very quiet.	
158	11	33	E. B. Ft.	20	21½	Ft. Eng.	..	23.0	0.62	0.80	0.10	6.2	....	13.4	....	4.6	10.0	....	0.20		
..	11	..	....	..	....	....	..	....	2.35	0.75	0.22	14.9	10.0	10.6	9.4	....	....	....	0.18	Slightly cumulative.	
..	11	..	....	..	....	....	..	....	3.40	0.67	0.10	....	....	10.0	....	....	....	6.2	0.20	"	"
159	11	34	W. B. Ft.	..	....	Ft. Eng.	..	....	0.44	0.80	0.25	15.6	Abt. 11.5	....	....	2.0	12.5	....	0.35		
160	11	35	E. B. "	11	11.	Std. 48' 7"	55	31.0	0.42	0.72	0.05	3.4	....	13.8	....	2.0	12.4	....	....		
..	11	..	....	..	....	....	..	....	0.70	0.50	0.05	5.0	....	12.3	....	....	....	....	....	Cumulative.	
..	11	..	....	..	....	....	..	....	0.90	0.50	0.25	25.0	Average 12.5	Abt. 13.5	12.7	....	....	....	....	"	
..	11	..	....	..	....	....	..	....	1.20	0.38	0.20	26.3								"	
..	11	..	....	..	....	....	..	....	1.70	0.40	0.25	31.2								"	
..	11	..	....	..	....	....	..	....	2.00	0.27	0.05	9.1	....	14.8	....	....	....	....	....	Load rapidly decreasing.	
..	11	..	....	..	....	....	..	....	2.45	0.00	Small.	....	21.3	22.0	....	....	....	....	....		
162	11	37	E. B. Pass.	10	....	Pass Eng.	..	....	0.38	0.80	0.22	14.1	Abt. 12.6	13.4	....	2.7	14.0	....	0.40	Cumulative.	Irregular.
164	11	39	E. B. Ft.	33	34.	Condn. 50'	48	23.9	0.57	0.95	0.17	9.2	....	12.7	....	7.2	11.1	....	0.16	Sinuosities broken.	
..	11	..	....	..	....	....	..	....	0.98	0.65	0.07	5.8	10.3	11.2	9.7	....	....	5.9	0.16	Cumulative. Load slowly	Regular.
..	11	..	....	..	....	....	..	....	2.70	0.85	0.35	20.6	10.0	10.1	9.7	....	....	....	0.10	increasing.	
..	11	..	....	..	....	....	..	....	3.27	0.45	0.02	2.8	....	12.7	9.7	....	....	....	0.10	do	
..	11	..	....	..	....	....	..	....	3.70	0.25	....	....	20.0	....	....	....	....	....	....		
..	11	..	....	..	....	....	..	....	6.50	0.72	0.05	3.4	....	10.7	9.7	....	....	....	0.20	Cumulative. Load rap-	
..	11	..	....	..	....	....	..	....	7.05	0.74	0.27	18.6	10.7	10.7	9.7	....	....	....	0.12	idly falls off and vibra-	
..	11	..	....	..	....	....	..	....	7.50	0.40	0.05	6.2	....	13.3	....	....	....	....	0.10	tion ceases.	
165	11	40	W. B. Pass.	4	....	Std. H. P. 49' 7"	68	....	0.38	0.87	0.25	14.3	12.0	13.0	....	1.3	11.0	5.0	0.38	Cumulative.	Regular.
167	11	42	W. B. Ft.	29	32.	Ft. Eng.	..	24.1	0.25	1.00	0.05	2.5	....	12.5	....	6.2	9. to 11.	....	....		
..	11	..	....	..	....	....	..	....	1.55	0.55	0.10	9.1	....	11.9	9.9	....	....	....	....	Cumulative, with load	
..	11	..	....	..	....	....	..	....	2.37	0.70	....	....	11.0	....	9.9	....	....	....	....	rapidly falling off,	
..	11	..	....	..	....	....	..	....	2.60	0.37	0.37	50.0	11.8	13.5	9.9	....	....	....	....		
..	11	..	....	..	....	....	..	....	3.02	0.30	0.07	12.5	....	14.4	9.9	....	....	....	....		
..	11	..	....	..	....	....	..	....	3.90	0.40	0.17	21.9	....	13.2	9.9	....	....	....	....		
..	11	..	....	..	....	....	..	....	6.60	0.00	....	....	....	....	....	....	....	....	....		
171	11	46	E, B. Pass.	4	....	Pass. Eng.	..	....	0.35	0.87	0.29	16.4	13.5 to 15.	13.0	....	1.1	14.0	....	....		
172	10	1	E B. "	5	....	"	..	39.8	0.33	0.72	0.27	19.0	15.0	14.0	....	1.3	15.5	....	0.25	Cumulative.	Sin. irregular
173	10	2	W. B. "	5	....	" 48'	66	40.8	0.37	0.77	0.35	22.6	13.3	13.6	....	1.3	13.5	5.5	0.45	"	Sin. regular.
176	10	5	E. B. Ft.	2	....	Std. Ft. 46'.	63	17.9	0.95	0.82	0.10	6.1	12.0	13.4	....	....	6.2	....	0.10	....	Sin. irregular.
..	10	..	....	..	....	....	..	....	2.48	....	Small.	....	22.5	24.5	....	....	....	....	....		
179	10	8	E. B. Pass.	5	....	Pass. Eng.	66?	45.1	0.29	0.75	0.20	13.3	17.0	13.8	....	1.1	15.0	....	0.30	Cumulative.	"
182	10	11	W. B. Pass.	5	....	"	66?	48.2	0.34	0.75	0.32	21.7	15.0	13.8	....	1.1	16.2	....	0.25	"	
183	10	12	W. B. Ft.	Abt. 21,	....	Std. Ft.	63	37.3	0.46	0.75	0.15	10.0	12.5	13.8	....	3.0	13.0	....	0.30	....	"
..	10	..	....	and 1	....	46'	..	....	1.50	0.42	0.15	17.6	13.5	14.2	....	....	....	6.5	0.15	Cumulative.	"
..	10	..	....	pay car.	....	....	..	....	3.65	....	Small.	....	24.6	24.5	....	....	....	....	....		
185	10	14	W. B. L. Ft.	10 ft.	....	Std. 47' 8"	63	43.0	0.38	0.67	0.15	11.1	13.3	14.3	....	1.7	15.0	....	0.25	....	"
..	10	..	....	and 1	....	....	..	....	1.58	0.30	0.12	20.8	29.0	15.8	15.4	....	....	6.0	0.12	Cumulative	
				coach.																	
186	10	15	W. B. Pass.	5	....	Pass. 48'.	66	51.1	0.30	0.75	0.30	20.0	16.0	13.8	....	1.1	17.0	....	0.25	"	
187	10	16	E. B. "	5	....	" Eng.	66?	55.3	0.22	0.74	0.20	13.6	17.7	13.9	....	1.0	18.4	....	0.32	"	
189	10	18	W. B. Ft.	24 ft.	....	Std Ft.	61	26.7	0.53	0.70	0.10	7.1	....	14.1	....	5.7	9.6	....	0.22	....	"
..	10	..	....	and 1	....	47'	..	....	2.60	0.25	0.10	20.0	18.0	16.7	9.6	....	....	6.2	0.04	Cumulative.	
..	10	..	....	coach.	....	....	..	....	3.65	0.61	0.05	....	....	....	....	....	....	....	0.15	....	"
192	10	21	W. B. Pass.	..	....	Pass. 48'	66	42.1	0.38	0.72	0.32	22.1	12.8	14.0	....	1.6	14.0	....	0.40	Cumulative.	"

* Speed for Nos. 18 to 39 inclusive, obtained by signals and measured base.
Speed for Nos. 129 to 172 inclusive, obtained from columns 3 and 15.
Speed for Nos. 173 and following, obtained from diameter and revolution of engine drivers.

† To agree with revolution of engine drivers, half-car lengths, panel lengths, etc., the vibrations are here given as "complete" or "double" vibrations, one of which includes two "simple" vibrations, or includes a forward and backward movement.

‡ Values given in this table were measured from the diagram. Multiply by 0.64 to obtain the actual movement of bridge in inches.

NOTE.—In column 2, for "W. B.," read "west bound," and for "E. B.," read "east bound." For "L. Ft.," read "local freight."
In column 5, for "Std. H. P.," read "standard heavy passenger engine," and for "Condn.," read "consolidation engine."
The percentages in column 11 were obtained by dividing half the amplitude of vibration by the statical deflection, thus giving the percentage of superadded deflection, or giving strain on bridge, as superadded by vibration, in percentage of statical strain due to live load.

TABLE No. 3.

Showing Cumulative Vibration of Railway Bridges from a Passing Engine, as Due to Counterbalance in Drivers and Distribution of Floor Beams

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Serial number.	Bridge (See Table No. 1.)	No. of Diagram.	Train. Passenger or freight.	Engine. Class and length, pilot to buffer.	Engine. Diam. of drivers.	Speed. Miles per hour.	Abscissa or distance from initial point of diagram.	Ordinate or statical deflection of bridge.*	Amplitude of vibration.*	Percentage of statical deflection superadded by vibration.	Observed number of complete vibrations per inch of diagram.	Number of floor beams passed per inch of diagram.	Number of revolutions of engine drivers per inch of diagram.	Remarks.
0	1		2	5	6	7	8	9	10	11	12	22	16	
					Inches.		Inches.	Inches.	Inches.	Per cent.				
18	2	4	E. B. Pass..	Pass. Eng............	66	....	0.32	0.70	0.40	28.6	14.5	....	....	
20	2	6	W. B. " ..	" 48'.........	66	46.5	0.30	0.60	0.28	23.3	15.5	17.9	16.2	
23	2	9	W. B. " ..	"	66	57.5	0.30	0.68	0.30	22.0	19.0	22.0	19.8	
27	3	4	W. B. " ..	"	66	46.6	0.30	0.73	0.26	17.8	16.0	18.0	16.3	Flooring very rigid.
29	3	6	W. B. " ..	" 48'.........	66	47.6	0.27	0.66	0.28	21.2	17.0	18.4	16.6	
32	3	9	W. B. " ..	"	66	46.2	0.30	0.81	0.30	18.5	16.0	17.2	15.4	
124	13	12	W. B. " ..	Std. H. Pass., 48'.....	66	....	0.55	0.83	0.38	23.4	......	....	....	
129	11	4	E. B. " ..	" 48'.....	66	23.1	0.37	0.82	0.30	18.2	abt. 13.0	12.2	11.0	
134	11	9	W. B. " ..	" 49' 7"..	68	34.5	0.35	0.89	0.29	16.2	13.3	12.7	11.2	
140	11	15	E. B. Fgt...	Consolidn., 50'.......	48	26.6	0.52	0.97	0.30	15.4	12.3	9.9	12.4	
141	11	16	E. B. Pass..	Pass. Eng............	....	35.9	0.27	0.87	0.31	17.8	13.5	....	....	
147	11	22	E B. " ..	Std. H. Pass., 49' 7"...	68	39.5	0.30	0.87	0.25	14.3	abt. 13.3	14.5	12.8	
151	11	26	E. B. " ..	" 48'	66	29.8	0.40	0.75	0.25	16.7	" 12.0	10.9	9.9	Flooring not rigid.
155	11	30	W. B. " ..	" 49' 7" ..	68	....	0.37	0.87	0.25	14.2	13.5	12.5	11.0	
159	11	34	W. B. Fgt..	Fgt. Eng.............	....	....	0.44	0.80	0.25	15.6	abt. 11.5	....	12.5	
162	11	37	E. B. Pass..	Pass. Eng............	....	....	0.38	0.80	0.22	14.1	" 12.6	....	14.0	
165	11	40	W. B. " ..	Std. H. Pas., 49' 7"...	68	....	0.38	0.87	0.25	14.3	12.0	18.2	16.0	
171	11	46	E. B. " ..	Pass. Eng............	....:	....	0.35	0.87	0.29	16.4	13.5 to 15	....	14.0	
172	10	1	E. B. " ..	"	66 ?	39.8	0.33	0.72	0.27	19.0	15.0	....	15.5	
173	10	2	W. B. " ..	" 48'.........	66	40.8	0.37	0.77	0.35	22.6	13.0	15.8	13.5	
179	10	8	E. B. " ..	"	66 ?	45.1?	0.29	0.75	0.20	13.3	17.0	17.5	15.0	
182	10	11	W. B. " ..	"	66 ?	48.2?	0.34	0.75	0.32	21.7	14.0	18.9	16.2	Flooring rigid.
186	10	15	W. B. " ..	" 48'.........	66	51.1	0.30	0.75	0.30	20.0	16.0	19.8	17.0	
187	10	16	E. B. " ..	"	66 ?	55.3	0.22	0.74	0.20	13.6	17.7	21.5	18.4	
192	10	21	W. B. " ..	" 48'.........	66	42.1	0.38	0.72	0.32	22.1	12.8	16.3	14.0	

* Values given in table were measured from diagram. Multiply by 0.64 to obtain the actual movement of bridge in inches.

TABLE No. 4.

Showing Cumulative Vibrations of Bridges for Passing Trains, as Due to the Speed, the Car, and Panel Length.

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Serial number.	Bridge (See Table No. 1.)	No. of Diagram.	Train, Passenger or Freight.	Number of Cars.	Number of Cars from Columns 12 and 15.	Length of Engine, Pilot to Buffer.	Speed, Miles per hour.	Abscissa or distance from initial point of diagram.	Ordinate or statical deflection of bridge.*	Amplitude of vibration.*	Percentage of statical deflection superadded by vibration.	Number of complete vibrations per inch of diagram. Observed.	Number of complete vibrations per inch of diagram. Calculated.	No. of half-car lengths per inch of diagram.	Inches of diagram per train.	Remarks.
0	1		2	3	4	5	7	8	9	10	11	12	13	14	15	19
			Freight.			Ft. In.		In.	In.	In.	Per Ct.					
22	2	8	W. B. Lo.	10 box	10 box	....	28.8	0.60	0.72	0.14	9.7					Cumulative shown on diagram as
..	..	..	......	and 1	and 1	....	....	1.00	0.54	0.30	29.8	12½ or 13.	....	12.5	..	immediately following the en-
..	..	..	......	coach.	coach.	....	....	1.50	0.40	0.10	12.5					gine.
61	6	6	E. B.	8	....	....	....	0.40	1.40	0.32	11.4	13.5	....	13.5	1.4	do.
..	..	..	......	....	....	....	....	0.65	0.80	0.30	18.7	17.0	....	for box	..	do.
..	..	..	......	....	....	....	....	1.20	0.74	0.06	....	....	....	cars.	..	do.
98	11	4	W. B.	15	....	....	....	....	0.62	0.28	22.5	....	....	....	..	Cumulative.
103	11	9	W. B.	....	....	....	....	....	0.72	0.20	13.9	....	....	....	..	do. slightly.
107	11	13	E. B.	15	....	50 0	....	....	0.79	0.34	21.5	....	...	....	..	do.
137	11	12	E. B.	30	32	50 0	26.1	1.12	0.52	0.14	13.1	....	12.1	....	6.1	Cumulative, beginning at end of
..	..	..	......	....	....	....	....	2.30	0.72	0.46	33.2	11.0	10.7	10.7	..	3 33′ cars, where 14 28′ box-cars
..	..	..	......	...	....	....	....	2.90	0.65	0.25	19.2	11.0	11.2	for box.	..	begin, and break up at 2 33′
..	..	..	......	....	....	....	....	3.35	0.70	0.42	30.4	11.0	10.9	....	..	stock cars. See 137, on Plate VI.
..	..	..	......	....	....	....	....	3.88	0.70	0.03	1.8	....	10.9	9.1	..	do.
														for stk.		do.
..	..	..	......	....	....	....	....	6.77	0.00	Small	....	22.0	22.0		..	
140	11	15	E. B.	31	31	50 0	26.6	1.00	0.37	0.04	4.9	....	....		5.9	Cumulative. See 140, on Plate VI.
..	..	..	......	....	....	....	....	1.62	0.71	0.26	18.4	11.0	10.8	10.9	..	do.
..	..	..	......	....	....	....	....	2.10	0.57	0.06	5.3	....	....	for box	..	
..	..	..	......	....	....	....	....	2.55	0.62	0.27	22.0	11.0	11.3	do.	..	do.
..	..	..	......	....	....	....	....	3.65	0.53	0.05	4.6	....	....	....	..	
..	..	..	......	....	....	....	....	4.78	0.66	0.07	5.7	....	....	....	..	
..	..	..	......	....	....	....	....	5.68	0.75	0.46	30.8	10.7	10.6	do.	..	do.
..	..	..	......	....	....	....	....	6.25	0.65	0.12	9.6	....	....		..	do.
148	11	23	E. B.	10	10½	49 4	....	0.87	0.50	0.07	....	....	....		2.2	do.
..	..	..	......	....	....	....	....	1.75	0.52	0.25	23.8	11.1	12.1	10.6	..	do.
..	..	..	......	....	....	....	....	2.35	0.30	0.03	....	....	....	....	..	do.
160	11	35	E. B.	11	11	48 7	31.03	0.70	0.50	0.05	5.0	....	12.3	....	2.0	do.
..	..	..	......	....	....	....	....	0.90	0.50	0.25	25.0					
..	..	..	......	....	....	....	....	1.20	0.38	0.20	26.3	Average 12.5	to	12.7	..	Load slightly diminishing.
..	..	..	......	....	....	....	....	1.70	0.40	0.25	31.2					
..	..	..	......	....	....	...	....	2.00	0.27	0.05	9.1	....	14.8	....	..	Load diminishing rapidly.
..	..	..	......	....	....	....	...	2.45	0.00	Small.	....	....	22.0			
164	11	39	E. B.	33	34	50 0	23.88	0.98	0.65	0.08	5.8	10.3	11.2	....	7.2	Cumulative. See 164, on Plate V.
..	..	..	......	....	....	....	....	2.70	0.85	0.35	20.6	10.0	10.1	9.7	..	do.
..	..	..	......	....	....	....	....	3.27	0.45	0.02	2.8	....	12.7	....	..	do.
..	..	..	......	...	....	....	....	6.50	0.72	0.05	3.4	....	10.7	....	..	Cumulative. Load and vibration
..	..	..	......	....	....	....	....	7.05	0.74	0.27	18.6	10.7	10.7	9.7	..	soon diminish.
..	..	..	......	....	....	....	....	7.50	0.49	0.05	6.2	....	13.2	....	..	
167	11	42	W. B.	29	32	....	24.14	1.55	0.55	0.10	9.1	....	11.9	...	6.2	Cumulative. See 167, on Plate VII.
..	..	..	......	....	....	....	....	2.37	....	....	....	11.0	....	9.9	..	do.
..	..	..	......	....	....	....	....	2.60	0.37	0.37	50.0	11.8	13.5	9.9	..	do.
..	..	..	......	....	....	....	....	3.02	0.30	0.07	12.5	....	14.4	....	..	do.
..	..	..	......	....	....	....	....	3.90	0.40	0.18	21.9	....	13.2	....	..	Momentarily cumulative.
..	..	..	......	....	....	....	....	6.60	0.00							

* Values given in table were measured from diagram. Multiply by 0.64 to obtain the actual movement of bridge in inches.

TABLE No. 5.

SHOWING ACTUAL DEFLECTION AND TIME OF V

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Bridge. (See Table No. 1.)	Span.		Depth of Truss.		Weight of Bridge.	ENGINE CENTER ABOUT TEN FEET PAST MID-SPAN, FOLLOWED BY TRAIN.						BRIDGE
						Weight of Engine.	Weight of Tender.	Length of Engine and Tender.		Maximum statical deflection for train. Actual.	Observed time of one complete vertical vibration.	Statical deflection. Actual.
	Ft.	In.	Ft.	In.	Pounds.	Pounds.	Pounds.	Ft.	In.	Inches.	Sec.	Inches.
2	140	6	24	0	227 500	72 150	43 500	48	0	0.38	0.26	0.23
										0.43	0.22	0.34
										0.44	0.28	....
						72 150	43 500	48	0	0.46	0.41	....
3	156	0	24	0	269 259	72 150	43 500	48	0	0.42	0.24	0.24
										0.46	0.26	0.36
						72 150	43 500	48	0	0.48	0.29	0.42
5	128	3	16	6	....	....	....	......		0.37	....	0.53
										0.40	....	....
										0.40	....	....
6	128	0	15	0	....	....	....	......		0.86	0.52	0.34
										0.89	0.50	0.53
7	154	6	26	0	....	....	....	......		0.04	Very small.	....
										0.06		....
										0.06		....
8	135	0	17	6	....	....	....	......		1.66to2.0	....	....
9	150	0	24	0	....	....	....	......		0.45	....	0.25
										0.45	....	0.25
										0.55	....	0.44
										0.62	....	0.48
10	148	0	24	0	223 575	70 600	39 200	47	8	0.43	....	0.16
						72 150	43 500	48	0	0.46	0.31	0.18
										0.47	0.22	0.19
						72 150	43 500	48	0	0.48	0.24	0.25
						74 700	45 600	48	10	0.48	0.28	0.27
										0.48	0.23	0.38
						72 150	43 500	48	0	0.49	0.29	0.44
												0.51
11	141	0	Abt. 24′		174 426	....	....	48	7	0.46	....	0.16
								48	0	0.48	0.33	0.24
						73 000	45 000	49	4	0.48	....	0.24
								48	0	0.52	0.30	0.26
						73 000	45 000	49	4	0.53	....	0.33
						81 600	52 400	49	7	0.56	0.29	0.38
						81 600	52 400	49	7	9.56	0.29	0.40
						81 600	52 400	49	7	0.56	0.33	0.41
						81 600	52 400	49	7	0.56	0.29	0.43
						100 000	46 500	50	0	0.60	....	0.43
						100 000	46 500	50	0	0.62	....	0.43
						100 000	46 500	50	0	0.62	....	....
						100 000	46 500	50	0	0.62	0.32	0.44
								50	0	0.63	....	....
						100 000	46 500	50	0	0.71	....	0.45
												0.46
												0.47
												0.48
												0.50
												0.54

NOTE.—The bridge movements are given in this

No. 5.

E OF VIBRATION OF RAILWAY BRIDGES.

Bridge uniformly loaded with ordinary train.				Bridge unloaded.		
Statical deflection. Actual.	Observed time of one complete vertical vibration.	Calculated time of one complete vertical vibration.	Observed time of one complete lateral vibration.	Observed time of one complete vertical vibration.	Observed time of one complete lateral vibration.	Remarks on vertical vibration for uniform load, and for an engine followed by its train.
Inches.	Sec.	Sec.	Sec.	Sec.	Sec.	
0.23	....	...	0.87	Abt.0.18	0.55	Cumulative vibration observed for both engine and train.
0.34	0.32	....	....	....	0.55	
....	....	....	....	....	....	
....	....	....	....	....	....	
0.24	0.26	....	0.91	0.18	0.63	Cumulative only for engines of passenger trains. Observed freight trains are slow here.
0.36	0.39	....	0.69	0.16	0.69	
0.42	0.37	....	0.73	0.20	....	
0.53	....	....	0.77	....	0.50	Cumulative for engine.
....	....	....	....	....	0.47	
....	....	....	....	....	0.45	
0.34	0.30	....	....	0.23	....	Cumulative for engine and for train.
0.53	0.40	....	....	....	....	
....	Very small.	....	Very small.	Very small.	Very small.	No vibrations exceeding one-tenth of an inch. Bridge exceedingly stiff.
....		....				
....		....				
....	....	....	....	....	....	Extraordinary deflection, but no considerable vibration.
0.25	....	....	....	....	....	One in the seven observations shows slight cumulative vibration for engine.
0.25	....	....	....	....	....	
0.44	....	....	....	....	....	
0.48	....	....	....	....	....	
0.16	0.22	0.23	0.63	0.16	0.58	Vibrations cumulative for engine, though only slightly so for freight trains.
0.18	0.25	0.25	....	0.17	0.60	
0.19	0.13	0.25	0.65	....	....	
0.25	....	....	0.71	....	....	
0.27	0.29	0.28	0.60	....	....	
0.38	0.28	0.31	0.58	....	....	
0.44	0.33	0.33	Abt. 0.6	....	....	
0.51	....	....	0.98	....	....	
0.16	0.22	0.26	....	0.18	0.63	Remarkable cumulative vibration for freight trains, and none for passenger trains. Also cumulative for passenger engines, and rarely so for freight engines.
0.24	0.33	0.29	....	0.18	0.63	
0.24	0.31	0 29	....	0.18	0.58	
0.26	....	0.30	0.75	0.18	0.58	
0.33	0.35	0.32	....	....	....	
0.38	0.29	0.34	....	....	....	
0.40	0.36	0.35	....	....	....	
0.41	0.36	0.35	0.75	....	....	
0.43	....	0.35	0.78	....	....	
0.43	0.31	0.36	0.62	....	....	
0.43	0.33	0.36	0.65	....	....	
....	....	....	....	....	....	
0.44	....	0.36	0.65	....	....	
....	....	....	....	....	....	
0.45	0.35	0.36	0.68	....	....	
0.46	....	0.36	0.65	....	....	
0.47	0.37	0.37	....	....	....	
0.48	0.37	0.37	....	....	....	
0.50	....	0.37	0.83	....	....	
0.54	0.39	0.39	....	....	....	

. in this Table in actual values in inches.

The maximum observed percentage of superadded deflection occasioned by vibration is given at 28.6 for the engine heading a train, and 50 for the train. This may be regarded as expressing the superadded strain in percentage of the statical strain due to live load, and properly termed cumulative dynamic effect. In providing for these strains in designing bridges, the greater should be taken, unless it is found that the train load, plus 50 per cent., is less than the engine load, plus 28 per cent.

Assuming that the vibration is likely to occur with equal percentages to trains of all loads, then if the train load ever equals the engine load the 50 per cent. is to be taken. Referring to the diagrams, 82 makes the train load over 7 per cent. greater than the engine load, and in 158 it is slightly greater. In a number of other diagrams it is fully up to the engine load; from which it appears that the higher of the two percentages must be adopted.

But, relative to the specific percentage, it is evident that the highest ever likely to occur in the lifetime of a bridge should be provided for in that bridge, and it is not likely that in watching a bridge a week or month under an indicator the highest possible percentage will be caught, because it has already been shown to be of unknown limit, unless it be that which would jump the bridge from its seat, and this, for a span of 150 feet, would be at about 150 per cent., or only about three times as great as the highest already observed.

But, if we allow the 50 per cent. for cumulative dynamic effect, then the factor of safety might be correspondingly changed, the percentage of which should be determined with due regard to the dead load as well as live load. This done, the working strength of iron might be raised from 10 000 pounds per square inch to about 13 000 pounds; or, allowing for 100 per cent., to about 16 000 pounds per square inch for spans of 150 feet, since for such spans the strains due to the dead load, the live load, and the superadded percentage are about as 4 to 6 to 3 for the 50 per cent., and as 4 to 6 to 6 for the 100 per cent.

Taking 15 000 pounds per square inch as an admissible working strength, where all strains are accounted for, then, by so designing our bridges as to destroy the cumulative dynamic effect, a reduction in weight of bridge can be realized nearly in the ratio of 15 to 10, and all standing on a more rational basis.

For passenger trains no cumulative dynamic effect has been observed for the train itself, a fact probably due to the discordant relation of panel and car, and to the variation, 45 to 60 feet, in passenger-car lengths. The most favorable panel length for vibration that is likely to be found in existing bridges would seem to be about 18 feet, or a third of the average car length, for which the favorable speed would be about 40 miles per hour. But only two of the bridges indicated had panels near this length, and only three diagrams were obtained on which a

cumulative result might have appeared. As none were obtained, it is to be supposed that the conditions were not favorable.

To Reduce Cumulative Dynamic Effect.

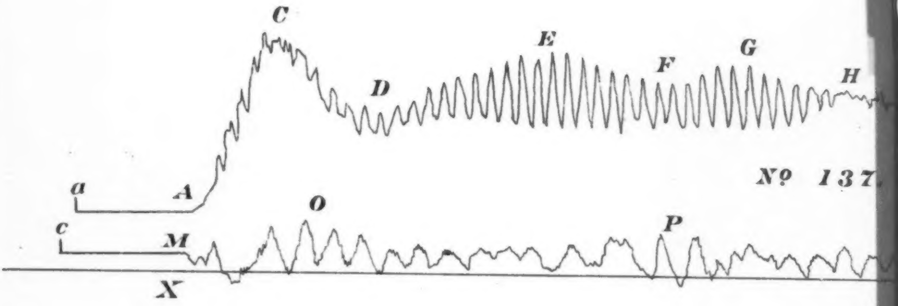
First.—For the locomotive it is necessary that the machinery be so counterbalanced as to occasion no vertical components of non-balance. This can readily be done, even as engines are now built; but it is not at all likely that it will be, because of the resulting excessive jerks then appearing as longitudinal components. This is due to the utter impossibility of perfectly counterbalancing the pistons of ordinary locomotives by counterweights in the wheels. In the Shaw locomotive we have an example of one free from the evil of non-balance.

Second.—If the bridge stringers could be perfectly rigid there would be nothing to fear from the train; but as this is impossible, let them be made very deep; not less than the depth of the floor beams, and then make the panel length so that no multiple of it will equal the length of any car, freight or passenger.

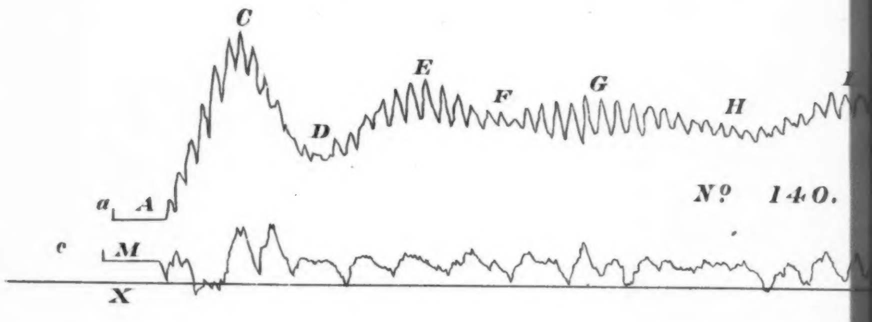
Third.—If expansion rollers at pedestal blocks be discontinued, and some provision be made for sliding, as upon "end wood" saturated with tallow, the "working strains" in the chords will probably be less than in use of perfectly free expansion rollers, and the greater the friction the less the chord strains, because, for either a rising or falling temperature, when a maximum train passes the bridge the lower chord will be elongated about a third of an inch per 100 feet, with free pedestals; and considerably less with severe friction, with corresponding strains. The endurance of abutments and piers is no part of the present problem. When the vibration is great enough to slip the pedestals back and forth against friction, the frictional resistance will largely counteract vibration. Thus for a span of 140 feet, maximum live and dead load 500 000 pounds, coefficient of friction 0.2, and vibration 50 per cent., the slip of pedestal is about a quarter of an inch for each vibration, and the foot pounds of resistance at each double vibration is about 2 084, and it will neutralize an equal applied impulse. At a vibration of about 30 per cent. the slipping begins. Hence substituting sliding for rolling serves a twofold purpose: first, to reduce direct chord strains; and second, to diminish vibration and the superadded strains.

Impact, Dynamic Effect, etc., Previously Provided For.

It has been customary to allow a diminishing percentage for impact, it being from 20 to 50 per cent. for short spans, and vanishing at spans of about 100 to 150 feet. But the cumulative dynamic effect will vary the opposite way with the span, and be 50 per cent. or more at 150 feet spans, where impact is neglected, from which it would appear that a



Nº 137.



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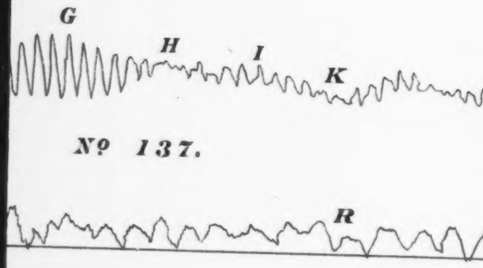

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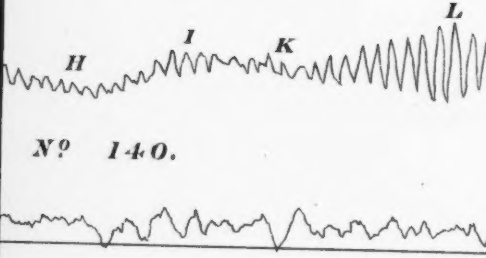

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PLATE VI
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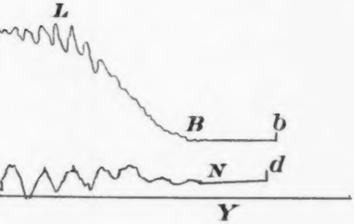
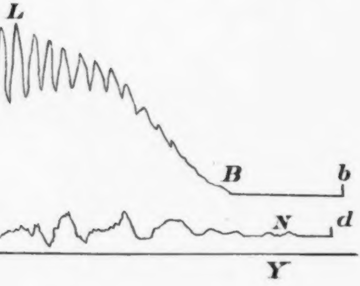
ROBINSON ON
VIBRATION OF BRIDGES.





constant allowance of 50 per cent. at least should be made for all spans, unless the trusses are so designed as to avoid vibration.

Influence of Style of Bridge on Vibration.

See remarks on "*Bridge Vibration and Oscillation*" on this point.

The Bollman, so far as observed, remains remarkably quiet under moving trains, while all Pratt trusses observed vibrated, though some but slightly. It is probable that all trusses with an upper and lower chord will vibrate, while others, like the Bollman, Fink, etc., will not.

General Conclusions.*

To avoid cumulative vibration in railroad bridges, it is essential:

First.—That the vertical component of non-balance of drivers be zero.

Second.—That the excess of non-balance of drivers be not down at mid-panel.

Third.—That the panel length and driver circumferences differ.

Fourth.—That the vibration of the engine on its springs, and of the bridge be discordant.

Fifth.—That the revolutions of drivers and complete vibrations per second be unequal.

Sixth.—That the bridge panel and half-car lengths be unlike.

Seventh.—That the number of panels and vibrations per second disagree for a passing train.

Eighth.—That the stringers be rigid.

Ninth.—That sliding be substituted for expansion rollers.

Tenth.—That pedestals be not at neutral axis of bridge; that the stringers may be laid out into banks for friction; that the car springs should be so proportioned that the times of car and bridge vibration differ, and that the car loading be not uniform.

Eleventh.—That the possible vibrations by the engine are limited by length of bridge, and those by the train by length of train, unless the bridge jumps out of its seats.

[Acknowledgments are due to Mr. C. F. Marvin, mechanical engineer, for assistance in the construction of an efficient indicator, and for the first application of it to bridges. Also to Mr. H. L. Wilgus, B.S., for continued use of instrument, and to bridges in the East. And also to Mr. E. O. Ackerman, civil engineer, for the remarkable diagrams obtained under his hand, and his valuable assistance in working out final results.]

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* For a more extended discussion of some of these conclusions, and the bearings of the same upon the adoption of unit working stresses for railway bridges, see Transactions of this Society, Vol. XV, June, 1886, page 432.

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TRANSACTIONS.

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THE WATER SUPPLY, DRAINAGE AND SEWERAGE OF THE LAWRENCEVILLE SCHOOL.

By FREDERICK S. ODELL, M. Am. Soc. C. E.

READ JUNE 16TH, 1886.*

The Lawrenceville School, on the John C. Green foundation, situated at Lawrenceville, New Jersey, about midway between Princeton and Trenton, is a high school for boys.

A school was established at this point in 1809 by the Rev. Dr. Brown, which soon attained considerable reputation, which was not diminished after its transfer in 1845 to the Rev. S. M. Hamill, who conducted it until 1882, when the property was purchased by the Trustees of the large fund devised for educational purposes by the Hon. John C. Green, of Trenton.

A considerable portion of this fund has been devoted to the enlargement of the facilities afforded by the College of New Jersey, and the purchase of the Lawrenceville Institution was intended mainly to furnish a thoroughly equipped preparatory school for the course at Princeton.

* Revised to February, 1887.

The ground occupied by the school is nearly a parallelogram, with a frontage of 1 000 feet on the old post road from Trenton to Princeton, and sloping southeastwardly for about 2 100 feet to a small brook, which flows through the property a few feet from its boundary, and about forty feet lower than the north corner of the tract.

The surface at the time of its purchase was in its natural state, slightly undulating, chiefly occupied as farming land, entirely without artificial drainage. The school buildings were huddled together near the road about the center of the frontage, and consisted of a variety of stone and frame buildings, added from time to time within the sixty years of the life of the school, as occasion demanded, with more regard to economy of construction and convenience of access than to architectural effect. The water supply was drawn from wells on the premises, and the privies were of the ordinary rural type.

The trustees decided to bring the institution more into conformity with modern ideas, and secured the services of Mr. Frederick Law Olmsted to lay out the grounds; of Messrs. Peabody and Stearns to design the buildings; and of Mr. J. J. R. Croes, M. Am. Soc. C. E., to design the plans for the water supply and sewerage of the establishment and to construct the works for that purpose and also the roads. The writer was in charge of this construction.

The design of Mr. Olmsted comprises an elliptical lawn, of about 600 by 400 feet, with the main school building at the southern extremity of its longer axis, and with six masters' houses surrounding the ellipse, all facing the south and also looking on the lawn and towards the school building, which, being a memorial building of the founder, is the most prominent feature in the design, and is a very handsome and striking example of the Elizabethan Gothic style of architecture in brown sandstone. The houses are of pressed brick, and they are each to be occupied by the family of a master and a limited number of students. The house of the head master stands apart and nearer the road and the entrance to the grounds. Plate XIV shows the general plan.

Two of these houses are not to be built at present. The other six have been erected, and are now occupied.

There have also been erected on the grounds a gymnasium, a bath house, a boiler-house and laundry, and one of the old school buildings is retained as a dormitory and for business offices.

The arrangement, dimensions and character of occupancy of the

buildings being determined by the landscape architect and the architects, the engineer was required to provide for the furnishing of a water supply and the removal of household wastes and the rain water, all of which it was desirable should be accomplished within the limits of the property owned by the school.

The water of the brook which traverses the property is not suitable for domestic use. The course of the stream is through cultivated lands, generally highly manured in the spring, and the flow is very small in the summer and exceedingly irregular at any time. It is a "quick" watershed, and after even moderate rainfalls the water is very turbid. As a surface supply under these circumstances was not practicable, for want of storage room and settling facilities, examinations were made looking towards a supply from the ground water.

In a swale near the south corner of the property there were indications of a supply from springs. Borings were made in that vicinity, and while they showed the presence of considerable water, analyses gave evidence of pollution from sewage, for which no good reason seemed to exist. This was very marked in the case of the boring which yielded the most copious supply.

This boring was in a slight depression towards the southeasterly portion of the grounds, and it was found on examination that near the beginning of this swale, and more than five hundred feet from the well, was the outlet of an old drain conveying the waste water from the kitchen and laundry of the old dormitory building; this explained the mystery of the contamination of the water in this well.

Further experiment with this well established the fact that when large draughts were made upon it, the impurities greatly lessened, and finally, after three days continuous pumping, a sample of water was taken that gave a very good analysis.

As it was thought to be proven that a well at this point would yield a sufficient supply of good potable water when the evident source of contamination should be removed, that location was selected for a well for a permanent water supply. A thorough system of subsoil drainage was put in, diverting into the brook all water from the surface and the upper strata of the soil, and eighteen months later samples of water taken from the same boring gave an excellent analysis, and it has ever since continued to be of very good quality.

WELL.

A well 16 feet in diameter and 23 feet deep was sunk, the normal level of the water being 3 feet below the top. See Plate VIII.

The material encountered in the excavation was clay for 6 feet below the surface, and then loose brown sandstone which came out without blasting, and lay so loosely next the excavation that it became necessary to line the well with brick throughout; this was done, leaving weep holes at frequent intervals through the lower portion of the 8-inch brick lining, which was brought up to a dome at the top and keyed by the cast-iron frame of a man-hole cover 2 feet in diameter, the cover being perforated.

The water is drawn from the well by a Worthington steam pump in the boiler-house, 800 feet distant, through an 8-inch cast-iron suction pipe; the vertical pipe in the well being provided with a foot valve and air chamber. The suction pipe is laid level, and when the well is full the water flows to the pump by gravity.

The pumping-main is of cast-iron pipe of 6 inches diameter for 700 feet to the campus, where it branches, one line, of 6-inch pipe, following the line of the easterly buildings for 800 feet; the other branch, of 4-inch pipe, following the line of the westerly buildings and the north front of the property for 2 300 feet to a connection with the 6-inch line, where a 6-inch branch, 200 feet long, leads to the water tower. There is thus a complete circulation, whether the supply be taken from the tower or directly from the pump. There is a check valve near the pump and nine stop-cocks on the mains, with six fire hydrants so placed that with 400 feet of hose two streams of water can be thrown on any building.

The 6-inch pipes are 0.455 inch thick and weigh 29 pounds to the foot, and the 4-inch pipes are 0.393 thick and weigh 17 pounds to the foot.

It was part of the architects' design for the buildings that a library should be erected with a fine tower, and the engineer was desirous of utilizing this tower by placing in it a tank for water supply when the pump was not in operation. The Trustees, however, did not feel disposed to build a library at present, nor would they consent to putting a tank in the memorial building, nor were they willing to go to the expense of an ornamental structure for the water supply. The water tower erected, under instructions from them to make it as inexpensive as possible, is a plain cylindrical shaft of plate iron 10 feet in diameter and 85 feet high.

The plates are lapped at the joints, and riveted and caulked.

The vertical joints are lapped $3\frac{1}{2}$ inches and double riveted for 28 feet above the bottom, and other joints are lapped 2 inches and single riveted.

The plates are of $\frac{1}{4}$, $\frac{3}{16}$ and $\frac{1}{8}$-inch thickness, and the bottom is of $\frac{1}{4}$-inch plate.

The tower stands on a foundation of rubble masonry laid in cement-mortar, which is 17 feet in diameter and 8 feet deep.

Four lugs riveted to the tower are connected with wrought-iron bolts passing down through the masonry.

This tower is acknowledged to be, architecturally, a serious blemish in an otherwise harmonious and elegant design, but for this neither the architects nor the engineer are responsible.

The capacity of the well to furnish a full supply at all times was not considered by the designer of the works to be demonstrated with sufficient certainty to warrant the rejection of any auxiliary supply which might be found available. Moreover, the water from the sandstone, while not hard enough to be absolutely objectionable for steam-boilers and for the laundry, was not a very soft water. It was considered desirable therefore to utilize the rain-water which should fall on the 55 000 square feet of slate-roof surface of the buildings on the plateau. The collection of this water in a covered reservoir on the hill would, it was thought, serve three good purposes.

First, it would furnish a stored supply of good water for use in case of fire, or deficiency or temporary unfitness of the well-water. *Secondly*, it would furnish to the laundry and boiler-house, on the lower ground, a gravity supply of soft water; and *Thirdly*, it would afford a relieving reservoir to the drain pipes which should be laid to carry to the brook the rain water from the campus, which has an area of about 550 000 square feet.

The roof area being 10 per cent. of the total, while the area of the finished road surfaces is only about 10 per cent., the rest being in lawn, it was thought that in a heavy rainfall the subtraction from the drains of so large a proportion of the water which would reach them most quickly and certainly, without any loss from absorption, would warrant a reduction in the size of the outfall drain, which had to be carried about 1 700 feet to the brook.

The rain-water from the roofs was therefore led by an independent line of pipes to a reservoir built underground at a point near that building in the group which was nearest to the boiler-house.

The leaders from the roof are carried down to the depth of 4 feet underground, and the carriers to the reservoir are of 6, 8 and 10-inch vitrified salt-glazed stoneware pipe, laid true to line and grade, and the joints made with Portland cement.

The reservoir, plans of which are shown on Plate IX, is built in a pit excavated for the purpose. It is rectangular in shape, and the bottom is 16 feet below the surface of the ground. The interior dimensions of the reservoir are 36½ by 67 feet. The side walls are 18 inches thick, of rubble stone masonry laid in cement, and well backed against the sides of the excavation, which was in hard clay and soft rock.

Inside of the stone masonry is a 4-inch lining of brick with a 2-inch coating of cement-mortar between the brick lining and the stone wall, the whole being covered with brick arches in two spans of 17 feet 6 inches each. The bottom is of concrete with a surface coating of Portland cement. The interior surface of the brick lining was washed with a preparation of Castile soap and alum in solution to render it more impervious to water.

This preparation is the same which proved efficient on the walls of the Croton Reservoir Gate-house in 1862, and is fully described in the paper by Mr. W. L. Dearborn, Transactions American Society of Civil Engineers, Vol. I, p. 203.

There were used 10 pounds alum and 50 pounds Castile soap. Two coats of the preparation were applied. The surface coated was 2 700 square feet. The work took 18 days of labor of mason and his helpers, and cost $65, or about 2½ cents per square foot.

The capacity of this reservoir is 162 000 gallons, equivalent to nearly 5 inches of rainfall on the roofs which now feed it. It is provided with an overflow pipe to the pipes which carry the drainage of the roads and lawns.

In the reservoir is built a filter well, which consists of a circular 4-inch brick wall 9 feet in diameter, and inside of this is placed a 4-inch cast-iron suction pipe provided with a foot valve and an automatic air valve, and leading to the boiler-house 720 feet distant. The water from this reservoir was intended to be used in the laundry and for feeding the boilers, but can also be pumped directly into the mains and used in any emergency, such as a large fire, should other sources become exhausted.

When drawing from the filter well at the rate of 1 600 gallons an hour, the difference of level between the water in the reservoir and in the

filter well is 3 feet. The advantage of having this reserve of water was demonstrated on January 20, 1887, when a fire occurred in the dwelling of Dr. Hamill on land adjoining the school grounds, and a fire stream was kept up from the nearest hydrant to the burning building for eight hours. A hole was broken in the filter well, and the water of both the well and reservoir used. Since then an 8-inch stop valve has been put in the filter well, to enable a free supply to be drawn from the reservoir in such an emergency.

Drainage.

That portion of the grounds in which the buildings are located is generally dry and needed little subsoil drainage, but it was deemed advisable to lay subsoil drains near the buildings, and in three cases entirely around the foundation walls below the level of the cellar floors, so as to insure their being dry at all times. Subsoil drains were also laid along the drives and walks, and the entire play-ground was underdrained by parallel lines of subsoil drains laid 30 feet apart.

These subsoil drains are of round agricultural tile, from one and one-quarter to two inches in diameter, and are laid on uniform grades about three feet below the surface, and have their outlet in the nearest road basin.

To provide for the surface drainage of the drives and grounds, a complete system of drains was laid, following the general direction of the drives, with catch basins opening from the gutters at intervals of about three hundred feet. These drains are of salt-glazed stoneware pipe from 6 to 8 inches diameter, with joints of Portland cement-mortar. They are laid about three feet six inches below the surface, to true lines and grades, and have their outlet in the brook at the lower portion of the grounds.

At the time the outfall system was designed it was thought that the large extent of lawn surface on very flat slopes, and the deduction of the roof area from the water-shed, would so materially diminish the discharge from the rainfall, that a capacity of carrying off 100 cubic feet per minute, or about an inch and a half of water on the road surfaces per hour, would be sufficient.

The experience of the first six months of 1886 showed that this was not sufficient, as the road drains were overtaxed three times during that period, causing pools of water to be formed for over an hour in some

depressions of grade, and also causing the water to flow out through a man-hole on the lower level near the engine-house, and flood the boiler-room floor.

This was undoubtedly partly caused by two departures from the plans for constructing and operating the works.

First.—The side drainage of the road in front of the property was not completed according to the plans, and thus a large quantity of water flowed across the road and on the school grounds from an extended slope on the opposite side of the road.

Secondly.—The supply from the well having been plentiful, the steam engineer in charge of the boiler-house found it easier to draw all the water from that source than to open and shut the cocks which change the pump suction from the well to the rain-water reservoir, so that the latter was never used and all the roof water was discharged into the road drains at their connection with the outfall pipe.

But, even if due allowance is made for these irregularities, it is not unlikely that in the case of a heavy rainfall, when the ground on the campus is frozen, the capacity of the outfall would have been found to be too small. A direct connection has therefore been made between the junction of drains at the reservoir overflow-pipe and the pond, by a 12-inch pipe, making the total capacity of discharge 450 cubic feet a minute. The highway drains opposite to the school property have also been attended to, and the road water thus diverted from the grounds. So far (March, 1887) this has proved satisfactory in the heaviest rainfalls which have occurred since the pipe was laid, the rain-water reservoir having been used for the purpose for which it was intended, and the roof water consequently retained in it.

Sewerage.

The necessity of disposing of the sewage within a limited area of the grounds made it imperative that its volume be limited to a minimum, and therefore all surface or subsoil drainage was excluded from the sewers, and disposed of as previously related; then, to insure positive immunity from leaky joints, it was decided to use six-inch cast-iron pipe, with leaded joints, for the sewers.

The pipes were 0.395-inch thick, and weighed 25 pounds to the foot. They were coated with coal-tar varnish, as were all the cast-iron pipe used on the grounds. Details of castings are shown on Plate X.

There are two branch lines of sewers, with a flushing man-hole at the head of each. The lines of the sewers are selected to serve every building with as short house connections as possible, and all deflections are made by special curved pipe. A man-hole is placed at every change in line or grade, and access is had to the sewer through a tee at the bottom of the man-hole, and also at the junction of house connections with the main line where the Y branch has cast in connection with it a vertical tee, from which a pipe is carried up to the surface of the ground.

Any man-hole may be used for flushing purposes. The flushing and cleaning is done very effectually by using a "pill," or spherical hard-wood ball, 5½-inches in diameter. This has proved more effective than one of smaller size.

The two branch sewers unite near the rain-water reservoir, and continue to the boiler-house and laundry, near which is placed the sewage tank, in which the solid matter in the sewage is allowed time to deposit itself on the bottom, and the partially clarified liquid is retained until it is desirable to discharge it into the sub-surface tiles.

Sewage Disposal System.

The sewage tank is built of brick-work underground, and is in two sections. The first or retaining section is in duplicate, and contains six compartments, three in each set. Each compartment is sixty feet long, about three feet wide and four feet deep. See Plate XI.

The sewage flows into one end of the first compartment, passes along its whole length, and at the other end passes into the second compartment through a quarter-bend pipe, with the mouth turned down below the level of the outlet, to prevent scum on the surface of the liquid from passing over into the second compartment, through which the liquid passes to its further end, and in like manner into the third, at the further end of which it passes over a weir into the receiving chamber, which is circular in form, twenty-five feet in diameter and eight feet deep. From this it is pumped by a pulsometer pump as often as necessary. This chamber is ventilated by a pipe leading into the flue of the boiler-house chimney. It is intended that whenever solids collect in such quantities that the settling compartments require cleaning, the sewage shall be turned in the duplicate set, and the sludge removed from the first.

It is found that nearly all the solids are deposited very near the en-

trance in the first compartment, and to cause the deposit to be distributed more evenly over the bottom, the water in the first compartment has been siphoned into the receiving chamber two or three times within the past six months. The rapid subsidence of the water, and the flow of incoming sewage during this operation, distribute the solids over the bottom, and enable the compartment to be used longer without cleaning out than would be the case if this distribution were not made.

The pulsometer has been so arranged that by attaching a suction hose, the water in the settling tanks can be pumped out and carried 300 feet through a hose to farm land ploughed to receive it. In January, 1887, the tanks were thus emptied, and the sludge then removed by a farmer to whom it had been sold. There were about 300 cubic feet of sludge removed from the first section of each of the settling tanks.

The irrigation ground comprises about one and three quarters acres, in the lower part of the school grounds, between the boiler-house and the brook. It is still further limited in location by the dam and pond on the westerly side, and an adjoining owner on the easterly side. It is the lowest portion of the school property, is naturally wet, and that portion near the brook (before drainage) was swampy. Its selection was a matter of necessity, it being all the land available for this purpose.

The natural surface of the ground was on a quite uniform slope from the higher portion to the brook, so that very little surface grading was necessary, but its thorough subsoil drainage became of the greatest importance.

To accomplish this, parallel lines of 2-inch round agricultural tile were laid, 40 feet apart, discharging into the brook.

These drains were laid 4 feet below the surface wherever the elevation of the brook permitted this depth; but, by reason of the elevation of the brook, the lower part of the drains were not deeper than from 2 to 2½ feet, and probably the average depth is not greater than 3 feet.

These drains were effective in drying the ground and preparing it to receive the sewage.

The distributing or sub-surface tiles were laid about eight inches below the surface, in nearly parallel lines 5 feet apart, on uniform grades of 9 to 12 inches in 100 feet. See Plate XII.

They are 2 inches in diameter and in 12-inch lengths.

They are laid on bed pieces of the same material and length, which

cover the bottom joints. Smaller pieces cover the top joint, leaving an opening on each side of $\frac{3}{4}$ by $\frac{1}{8}$ inch, out of which the water escapes into the soil.

The water enters these lines of sub-surface drains from a 4-inch carrier leading from a chamber into which the pulsometer discharges, and in which are the two 4-inch carrier pipes leading to different parts of the ground, into either of which the sewage can be turned at pleasure and the two sections of the field used alternately.

A special branch joins the 2-inch distributing tile with the 4-inch carrier, the 2-inch tile being so attached that its bottom is at the same level as that of the carrier from which it branches, so that if but little sewage is flowing in the carrier each line of drain will get its share, those in the upper portion of the field being prevented from surcharge by either flattening the grade or throttling the first section of drain.

There are about six hundred feet of 4-inch carrier pipe, and about twenty thousand feet of 2-inch drains on the $1\frac{3}{4}$ acre of ground.

The amount of sewage water averages 6 000 gallons a day.

This is discharged into the irrigation tile eight times in a month, or from 20 000 to 25 000 gallons at a time. The discharge from the outfall drains begins very soon after the tile are charged, showing the ground to be very porous.

No complaint has been made of any offensive odor or fouling of the stream.

The irrigation ground is not worked to nearly its capacity, as it has been found that the sewage does not flush the tiles fully to the lower extremity of the lines, and while the growth of the grass on the upper end of the lines is luxurious and rapid, the ground over the further end has remained bare or with very scanty vegetation.

Dam and Pond.

A small pond for bathing in summer, and skating and supplying ice in winter, had been connected with the school for some years, and was enlarged by building a dam further down the stream, taking material for it from the excavation of the pond. See Plate XIII.

The dam was made of earth laid in 6-inch layers, each sprinkled and rolled. The slopes are $2\frac{1}{2}$ to 1 on the water side, and 2 to 1 on the lower, though the lower slope was afterwards made much less steep by the ad-

dition of surplus filling, which was trimmed to an ogee curve and prepared by a coating of top soil for seeding.

A masonry overflow with wing walls is provided near the center of the dam, and a rubble masonry heart wall laid in cement rises within the embankment to the flow line.

The whole work, except the water-tower, was done by the contractors who erected the buildings, they receiving for profit a certain percentage of the cost.

Great care was exercised in laying all pipes to true lines and grades, and in making good substantial joints in both iron and stone-ware pipes.

The cost of laying the pipes was as follows.

The prices given include lead and gaskets for cast-iron pipes, and Portland cement and oakum gaskets for stone-ware pipes, and also a profit of ten per cent.

8-inch	cast-iron pipe	$0.22	per foot.
6 "	"	0.13½	"
4 "	"	0.10	"
10 "	stone-ware	0.04½	"
8 "	"	0.03½	"
6 "	"	0.03	"

The cost of the following structures is made up from accounts kept during construction:

Water-tower (including foundation)	$2 100
Well	1 400
Rain-water reservoir	4 450
Sewage tank	2 900
Irrigation grounds	2 000

With the exception of the occasional deficiency in the capacity of the rain-water drains above mentioned, the operation of the works during the year has been very satisfactory.

The regular number of persons now using the water and contributing to the sewage is 180. The works are designed to accommodate 400 people.

The water supplied for all purposes averaged 8 000 gallons a day in 1886, varying from 6 000 gallons a day in April to 25 000 gallons a day during one week in October, 1886, when the lawns were very dry and a new sprinkling cart was put in use on the roads and lawns.

The amount of water and sewage pumped in each month since the works went into operation has been as follows :

Month.	Gallons Pumped. Water.	Sewage.
1885.—October	230 957	178 364
November	348 824	199 227
December	277 475	151 050
1886.—January	199 600	166 500
February	259 100	178 000
March	250 000	185 500
April	181 500	165 000
May	255 000	191 000
June	204 000	189 000
July	184 000	43 000
*August	50 000	23 000
September	172 550	117 000
October	411 000	200 000
November	206 000	191 000
December	157 700	144 000
1887.—January	228 500	180 000
February	175 000	168 000

* Vacation.

PLATE VIII
TRANS. AM. SOC. CIV. ENG'RS.
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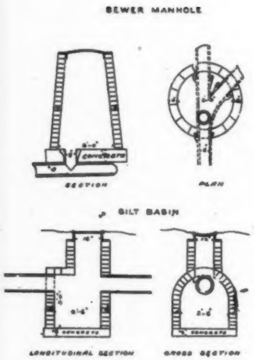
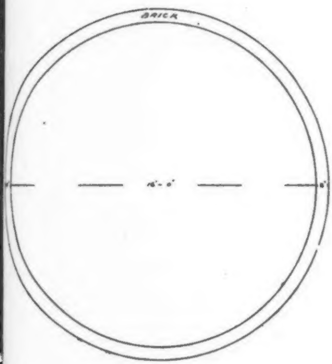
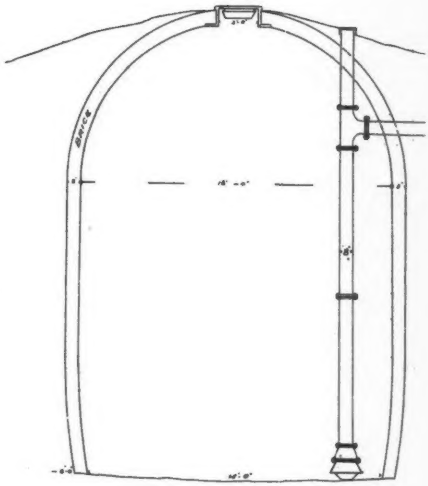
ODELL ON
LAWRENCEVILLE SCHOOL.



LAWRENCEVILLE SCHOOL
PLAN OF WELL
FOR WATER-SUPPLY



PLAN AT SPRING LINE



SECTION

PLATE IX
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ODELL ON
LAWRENCEVILLE SCHOOL.

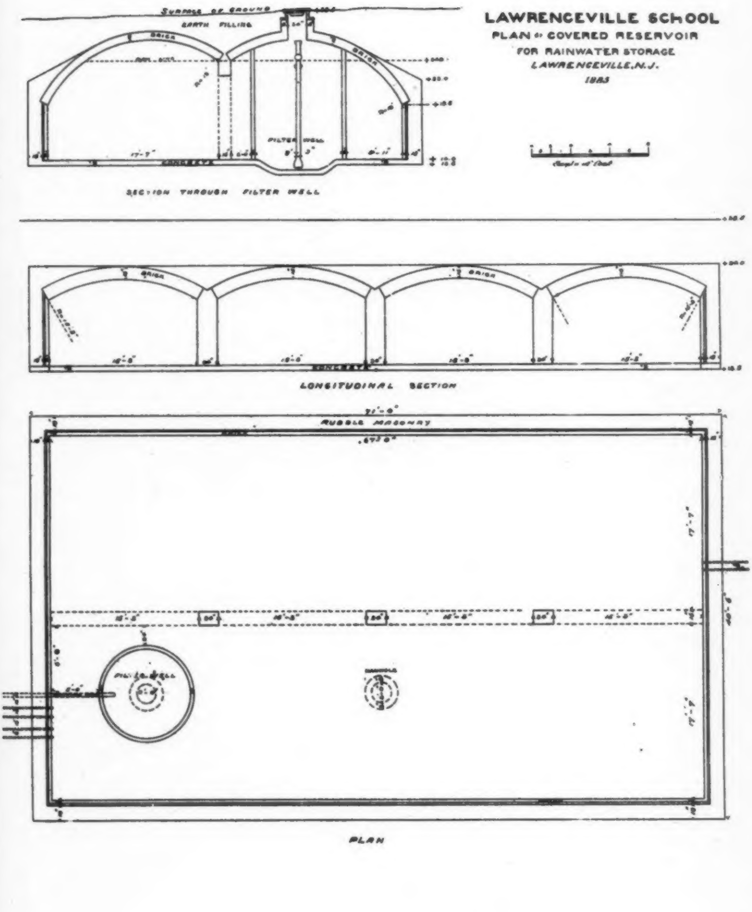

PLATE X
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LAWRENCEVILLE SCHOOL.

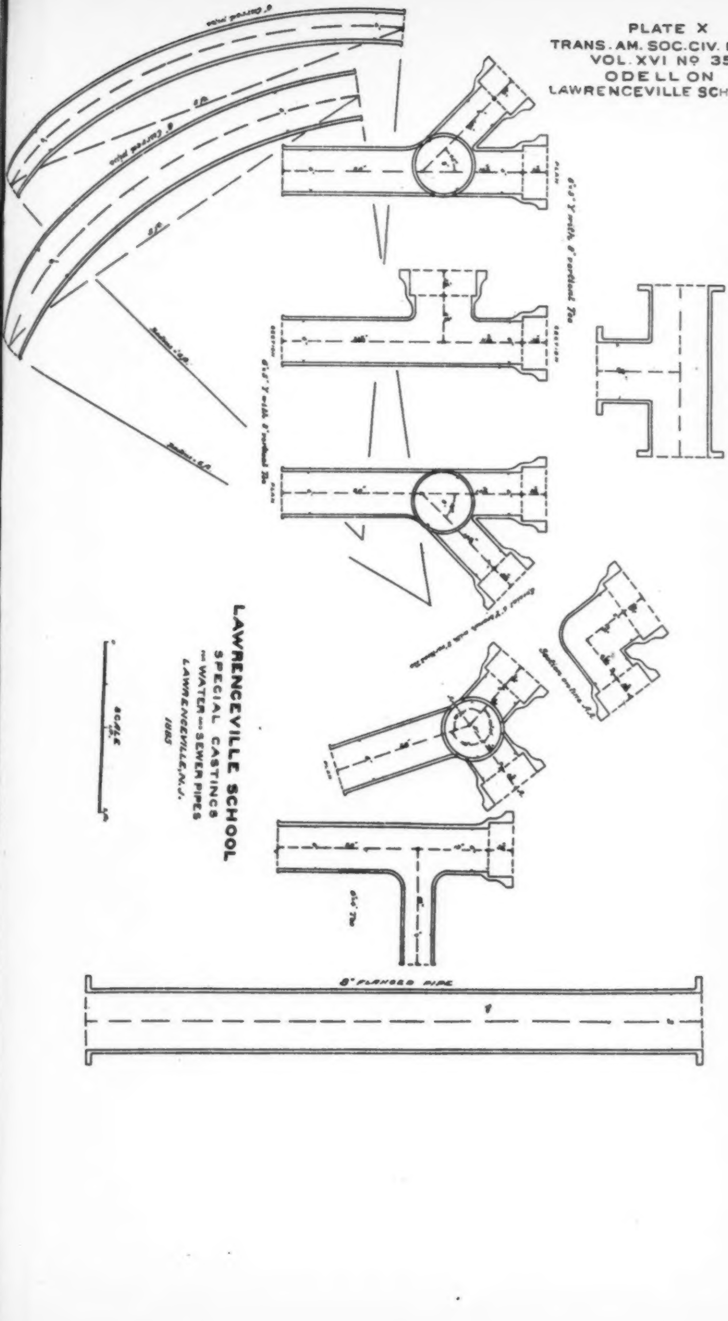

PLATE XI
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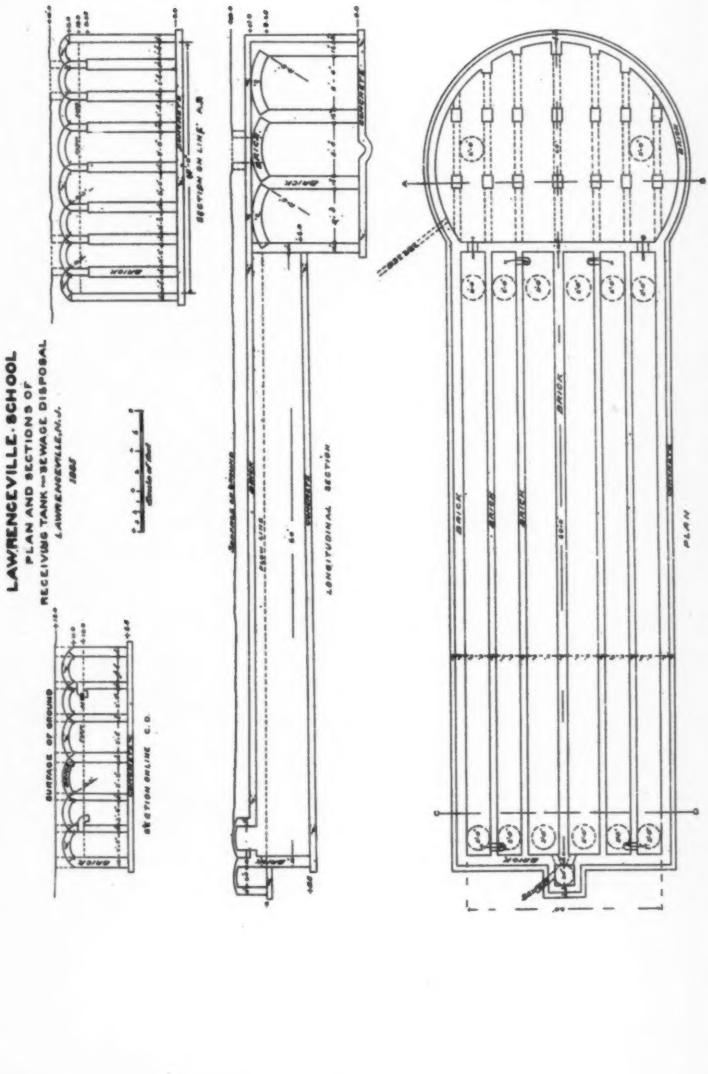

PLATE XII
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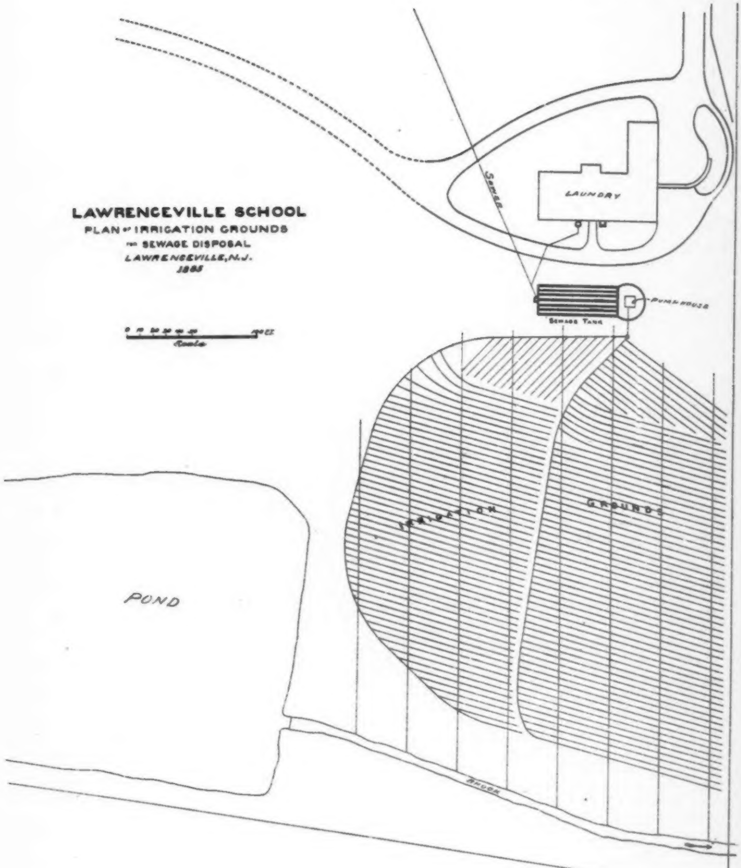

PLATE XIII
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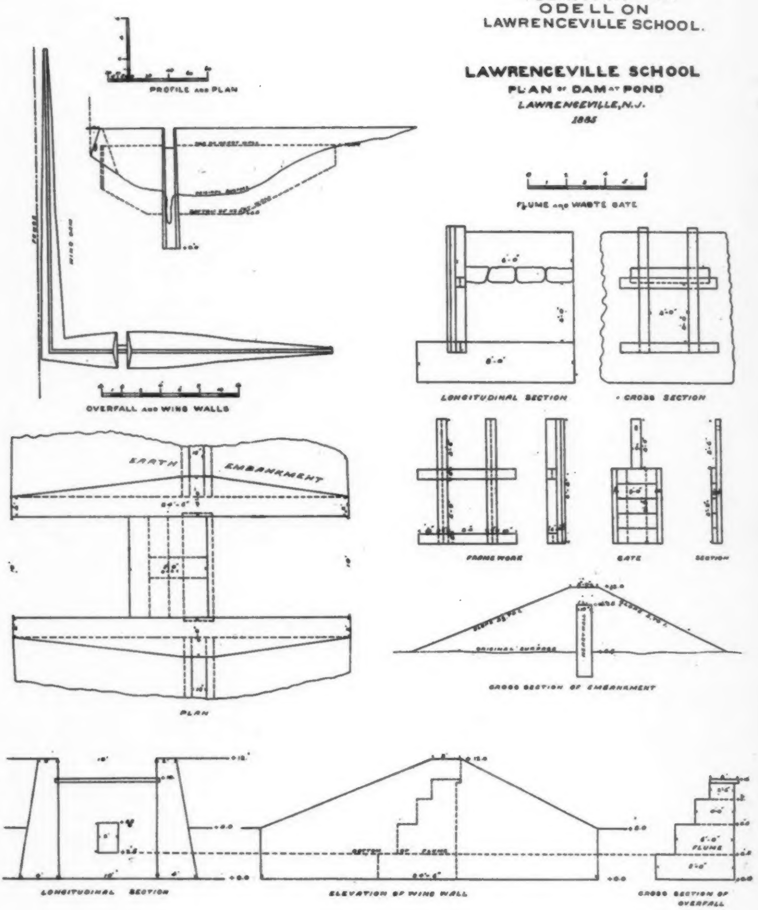
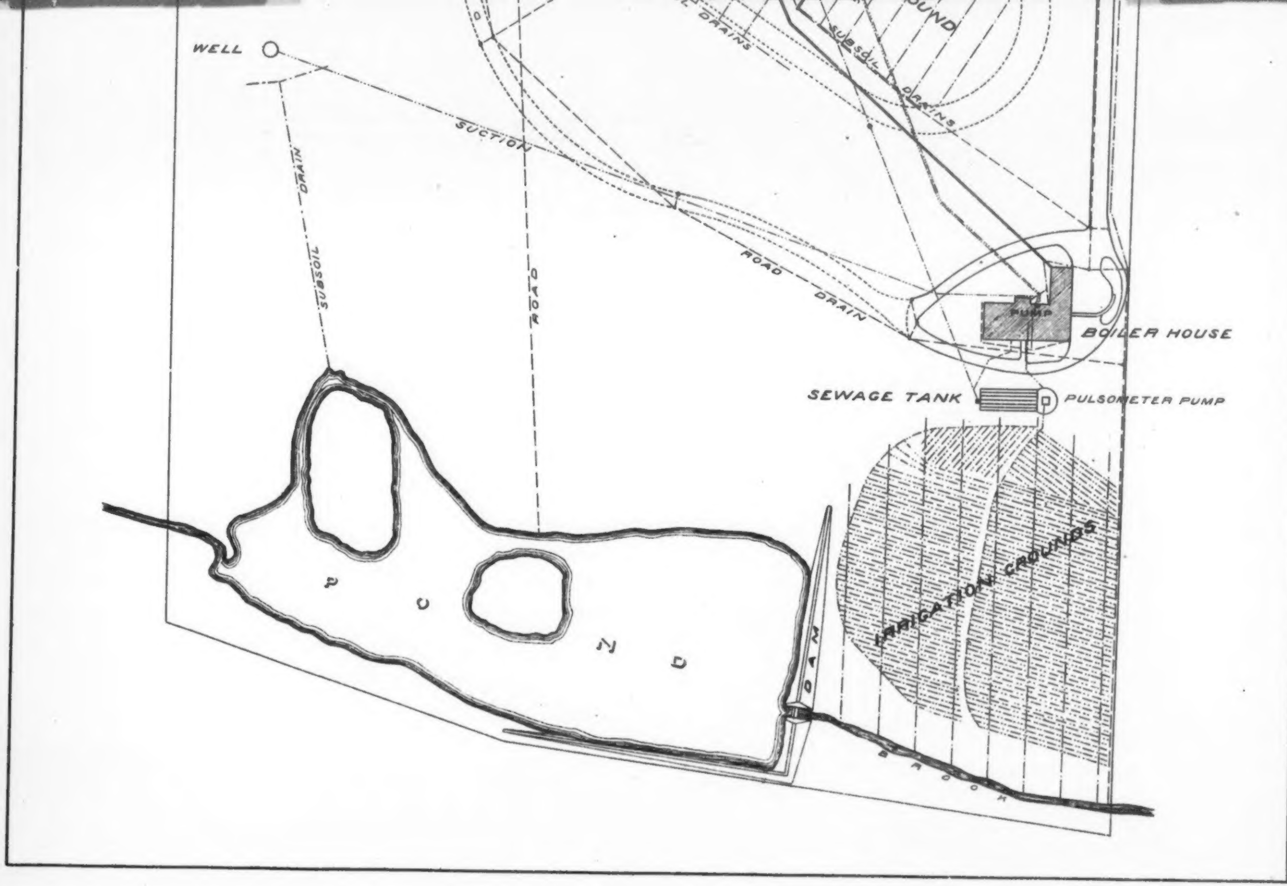
ODELL ON
LAWRENCEVILLE SCHOOL.

LAWRENCEVILLE SCHOOL
PLAN OF DAM AT POND
LAWRENCEVILLE, N.J.
1885




WELL
SUBSOIL DRAIN
SUCTION
ROAD
ROAD DRAIN
SUBSOIL DRAINS
PUMP
BOILER HOUSE
SEWAGE TANK
PULSOMETER PUMP
IRRIGATION GROUNDS
POND
DAM
BROOK

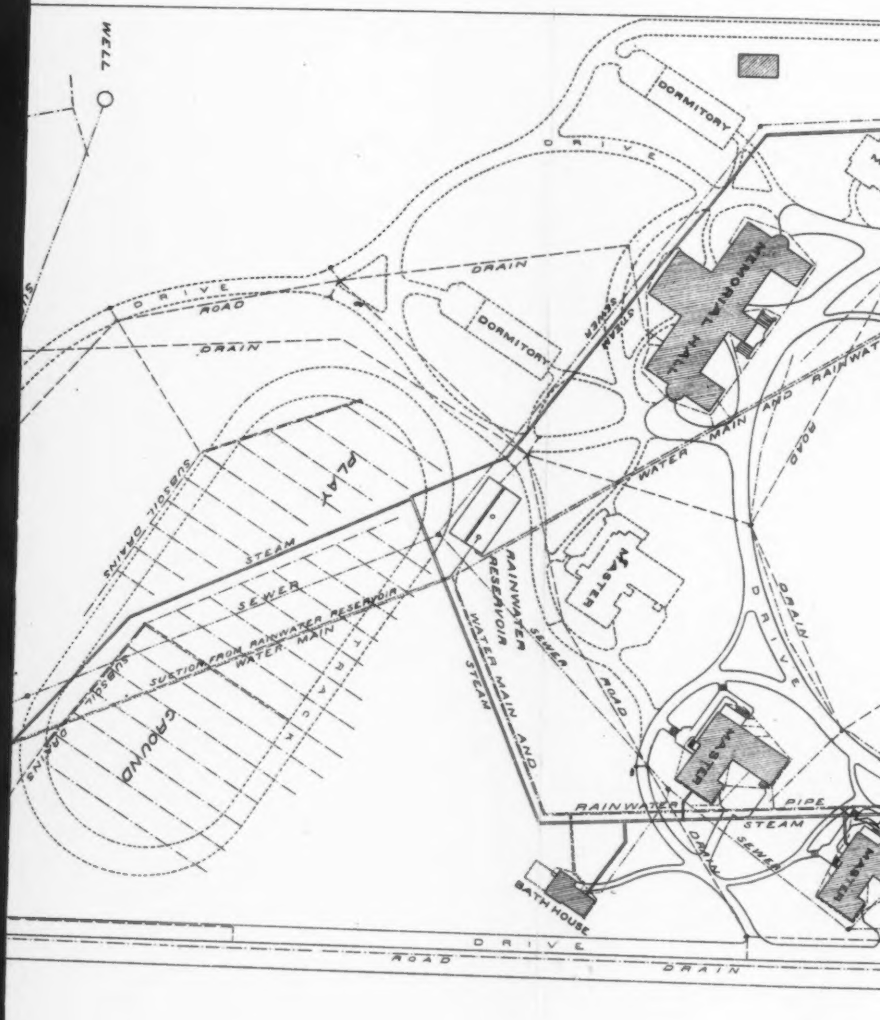

WELL
DORMITORY
DRIVE
DRAIN
ROAD
DRAIN
DORMITORY
MEMORIAL HALL
SEWER
STEAM
RAINWATER
WATER MAIN AND RAINWATER
ROAD
PLAY
GROUND
SUBSOIL DRAINS
STEAM
SEWER
SUCTION FROM RAINWATER RESERVOIR
WATER MAIN
TRACK
RAINWATER RESERVOIR
WATER MAIN AND STEAM
SEWER
ROAD
MASTER
DRAIN
DRIVE
MASTER
RAINWATER
PIPE
STEAM
DRAIN
SEWER
MASTER
BATH HOUSE
DRIVE
ROAD
DRAIN

PLATE XIV
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